



20. Positive integers x and y satisfy the equation $\sqrt{x} - \sqrt{11} = \sqrt{y}$.

What is the maximum possible value of $\frac{x}{y}$?

A 2

B 4

C 8

D 11

E 44

1190



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20. **B** Squaring the equation $\sqrt{x} - \sqrt{11} = \sqrt{y}$ gives $x - 2\sqrt{11x} + 11 = y$... (1). You see here that $2\sqrt{11x}$ is an integer. Thus $x = 11a^2$ for some integer a . Hence in (1), $y = 11a^2 - 22a + 11 = 11(a^2 - 2a + 1)$. Thus $\frac{x}{y} = \left(\frac{a}{a-1}\right)^2$ whose maximum value, for integer a , is easily seen to be $\left(\frac{2}{1}\right)^2 = 4$.