



24. Three spheres of radius 1 are placed on a horizontal table and inside a vertical hollow cylinder of height 2 units which is just large enough to surround them. What fraction of the internal volume of the cylinder is occupied by the spheres?

A  $\frac{2}{7 + 4\sqrt{3}}$       B  $\frac{2}{2 + \sqrt{3}}$       C  $\frac{1}{3}$       D  $\frac{3}{2 + \sqrt{3}}$       E  $\frac{6}{7 + 4\sqrt{3}}$



24. E The volume of the three spheres is  $3 \times \frac{4}{3}\pi \times 1^3 = 4\pi$ .  
Let  $r$  be the radius of the cross-sectional area of the cylinder.  
Hence the volume of the cylinder is  $2\pi r^2$ .

Thus the required fraction is  $\frac{2}{r^2}$ .

The straight lines joining the centres of the three spheres form an equilateral triangle of side length 2.

Let  $x$  be the distance from the centre of a sphere to the midpoint of the triangle. Using the

Sine Rule,  $\frac{2}{\sin 120^\circ} = \frac{x}{\sin 30^\circ}$  hence  $x = \frac{2}{\sqrt{3}}$ .

As the sphere has radius 1,  $r = x + 1$  and  $r = 1 + \frac{2}{\sqrt{3}}$ .

Thus  $r^2 = \frac{1}{3}(2 + \sqrt{3})^2 = \frac{1}{3}(7 + 4\sqrt{3})$ . Hence the required fraction is  $\frac{6}{7 + 4\sqrt{3}}$ .

