25. What is the area of the polygon formed by all points (x, y) in the plane satisfying the inequality $\big||x| - 2\big| + \big||y| - 2\big| \leqslant 4$?

A 24 B 32 C 64 D 96 E 112

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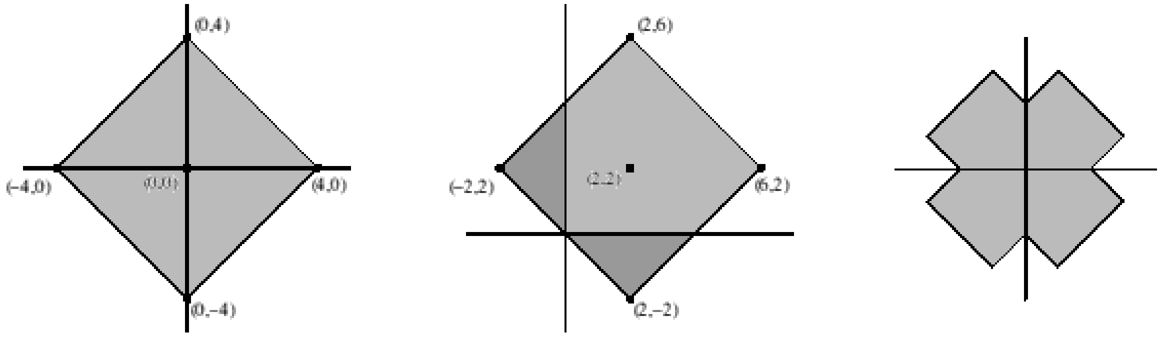
25. **D** To work out the area of $\big||x| - 2\big| + \big||y| - 2\big| \leqslant 4$, we first consider the region $|x| + |y| \leqslant 4$ which is shown in (a). This region is then translated to give $|x - 2| + |y - 2| \leqslant 4$ as shown in (b).

By properties of the modulus, if the point (x, y) lies in the polygon, then so do $(x, -y)$, $(-x, y)$ and $(-x, -y)$. Thus $\big||x| - 2\big| + \big||y| - 2\big| \leqslant 4$ can be obtained from (b) by reflecting in the axes and the origin, as shown in (c).



(a) (b) (c)

Hence the required area is 4 times the area in the first quadrant. From (b), the required area in the first quadrant is the area of a square of side $4\sqrt{2}$ minus two triangles (cut off by the axes) which, combined, make up a square of side $2\sqrt{2}$. So the area in the first quadrant is $(4\sqrt{2})^2 - (2\sqrt{2})^2 = 32 - 8 = 24$.

Hence the area of the polygon is $4 \times 24 = 96$ square units.