



22. Given that  $S = (x + 20) + (x + 21) + (x + 22) + \dots + (x + 100)$ , where  $x$  is a positive integer, what is the smallest value of  $x$  such that  $S$  is a perfect square?
- A 1                      B 2                      C 4                      D 8                      E 64

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22. C There are 81 terms in the series, so, using the formula  $S = \frac{1}{2}n(a + l)$  for an arithmetic series:

$$S = \frac{81}{2}(x + 20 + x + 100) = 81(x + 60).$$

Now 81 is a perfect square, so  $S$  is a perfect square if and only if  $x + 60$  is a perfect square. As  $x$  is a positive integer, the smallest possible value of  $x$  is 4.