



21. What is the sum of the values of  $n$  for which both  $n$  and  $\frac{n^2 - 9}{n - 1}$  are integers?
- A -8                      B -4                      C 0                      D 4                      E 8

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21. E Note that  $n^2 - 1$  is divisible by  $n - 1$ . Thus:

$$\frac{n^2 - 9}{n - 1} = \frac{n^2 - 1}{n - 1} - \frac{8}{n - 1} = n + 1 - \frac{8}{n - 1} \quad (n \neq 1).$$

So, if  $n$  is an integer, then  $\frac{n^2 - 9}{n - 1}$  is an integer if and only if  $n - 1$  divides exactly into 8.

The possible values of  $n - 1$  are  $-8, -4, -2, -1, 1, 2, 4, 8$ , so  $n$  is  $-7, -3, -1, 0, 2, 3, 5, 9$ .

The sum of these values is 8.

(Note that the sum of the 8 values of  $n - 1$  is clearly 0, so the sum of the 8 values of  $n$  is 8.)