An $n \times n$ square array contains 0s and 1s. Such a square is given below with $n = 3$.

$$\begin{matrix} 0 & 0 & 1 \\ 1 & 0 & 0 \\ 1 & 1 & 0 \end{matrix}$$

Two types of operation C and R may be performed on such an array.

- The first operation C takes the first and second columns (on the left) and replaces them with a single column by comparing the two elements in each row as follows; if the two elements are the same the C replaces them with a 1, and if they differ C replaces them with a 0.
- The second operation R takes the first and second rows (from the top) and replaces them with a single row by comparing the two elements in each column as follows; if the two elements are the same the R replaces them with a 1, and if they differ R replaces them with a 0.

By way of example, the effects of performing R then C on the square above are given below.

$$\begin{matrix} 0 & 0 & 1 \\ 1 & 0 & 0 \\ 1 & 1 & 0 \end{matrix} \xrightarrow{R} \begin{matrix} 0 & 1 & 0 \\ 1 & 1 & 0 \end{matrix} \xrightarrow{C} \begin{matrix} 0 & 0 \\ 1 & 0 \end{matrix}$$

(a) If R then C are performed on a 2×2 array then only a single number (0 or 1) remains.

(i) Write down in the grids on the next page the eight 2×2 arrays which, when R then C are performed, produce a 1.

(ii) By grouping your answers accordingly, show that if $\begin{matrix} a & b \\ c & d \end{matrix}$ is amongst your answers to part (i) then so is $\begin{matrix} a & c \\ b & d \end{matrix}$.

Explain why this means that doing R then C on a 2×2 array produces the same answer as doing C first then R.

(b) Consider now a $n \times n$ square array containing 0s and 1s, and the effects of performing R then C or C then R on the square.

(i) Explain why the effect on the right $n - 2$ columns is the same whether the order is R then C or C then R. [This then also applies to the bottom $n - 2$ rows.]

(ii) Deduce that performing R then C on an $n \times n$ square produces the same result as performing C then R.