

Alice, Bob and Charlie are well-known expert logicians.

(i) The King places a hat on each of their heads. Each of the logicians can see the others' hats, but not his or her own.

The King says "Each of your hats is either black or white, but you don't all have the same colour hat".

All four are honest, and all trust one another.

The King now asks Alice "Do you know what colour your hat is?".

Alice says "Yes, it's white".

What colour are the others' hats? [Hint: think about how Alice can deduce that her hat is white.]

(ii) The King now changes some of the hats, and again says "Each of your hats is either black or white, but you don't all have the same colour hat". He now asks Alice "Do you know what colour your hat is?".

Alice replies "No"

Can Bob work out what colour his hat is? Explain your answer. [Hint: what can Bob deduce from the fact that Alice can't tell what colour her hat is?]

(iii) The King now changes some of the hats, and then says "Each of your hats is either black or white. At least one of you has a white hat."

He now asks them all "Do you know what colour your hat is?". They all simultaneously reply "No".

What can you deduce about the colour of their hats? Explain your answer.

(iv) He again asks "Do you know what colour your hat is?" Alice says "No", but Bob and Charlie both say "Yes" (all three answer simultaneously).

What colour are their hats? Explain your answer.