



A triangle  $ABC$  has sides  $BC, CA$  and  $AB$  of sides  $a, b$  and  $c$  respectively, and angles at  $A, B$  and  $C$  are  $\alpha, \beta$  and  $\gamma$  where  $0 \leq \alpha, \beta, \gamma \leq \frac{1}{2}\pi$ .

(i) Show that the area of  $ABC$  equals  $\frac{1}{2}bc \sin \alpha$ .

Deduce the sine rule

$$\frac{a}{\sin \alpha} = \frac{b}{\sin \beta} = \frac{c}{\sin \gamma}.$$

(ii) The points  $P, Q$  and  $R$  are respectively the feet of the perpendiculars from  $A$  to  $BC$ ,  $B$  to  $CA$ , and  $C$  to  $AB$  as shown.

Prove that

$$\text{Area of } PQR = (1 - \cos^2 \alpha - \cos^2 \beta - \cos^2 \gamma) \times (\text{Area of } ABC).$$

(iii) For what triangles  $ABC$ , with angles  $\alpha, \beta, \gamma$  as above, does the equation

$$\cos^2 \alpha + \cos^2 \beta + \cos^2 \gamma = 1$$

hold?