Core 1 Polynomials Questions
(From the Oxford MAT Tests)

For answers, see the MAT website

Specimen A, Question 1b:

B. The smallest value of the function

$$f(x) = 2x^3 - 9x^2 + 12x + 3$$

in the range $0 \le x \le 2$ is

(a) 1 (b) 3 (c) 5 (d) 7

Specimen A, Question 1d:

D. The equation $x^3 - 30x^2 + 108x - 104 = 0$

(a) no real roots;

(b) exactly one real root;

(c) three distinct real roots;

(d) a repeated root.

Specimen B, Question 2:

Suppose that the equation

$$x^4 + Ax^2 + B = \left(x^2 + ax + b\right)\left(x^2 - ax + b\right)$$

holds for all values of x.

(i) Find A and B in terms of a and b.

(ii) Use this information to find a factorization of the expression

$$x^4 - 20x^2 + 16$$

as a product of two quadratics in x.

(iii) Show that the four solutions of the equation

$$x^4 - 20x^2 + 16 = 0$$

can be written as $\pm\sqrt{7} \pm \sqrt{3}$.

2008, Question 1d:

D. When

$$1 + 3x + 5x^2 + 7x^3 + \cdots + 99x^{49}$$

is divided by $x - 1$ the remainder is

(a) 2000, (b) 2500, (c) 3000, (d) 3500.

2009, Question 1f:

F. The equation in x

$$3x^4 - 16x^3 + 18x^2 + k = 0$$

has four real solutions

(a) when $-27 < k < 5$;

(b) when $5 < k < 27$;

(c) when $-27 < k < -5$;

(d) when $-5 < k < 0$.

2009, Question 1i:

I. The polynomial

$$n^2x^{2n+3} - 25nx^{n+1} + 150x^7$$

has $x^2 - 1$ as a factor

(a) for no values of n;

(b) for $n = 10$ only;

(c) for $n = 15$ only;

(d) for $n = 10$ and $n = 15$ only.

2009, Question 1j:

J. The number of *pairs* of *positive integers* x, y which solve the equation

$$x^3 + 6x^2y + 12xy^2 + 8y^3 = 2^{30}$$

is

(a) 0, (b) 2^6, (c) $2^9 - 1$, (d) $2^{10} + 2$.

2010, Question 1h:

H. Given a positive integer n and a real number k, consider the following equation in x,

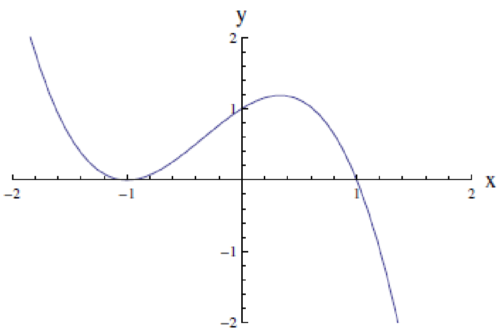
$$(x-1)(x-2)(x-3)\times\cdots\times(x-n)=k.$$

Which of the following statements about this equation is true?

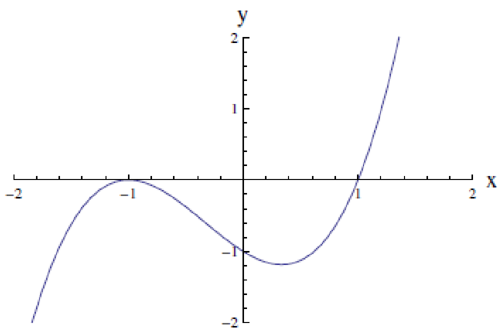
(a) If $n=3$, then the equation has no real solution x for some values of k.
(b) If n is even, then the equation has a real solution x for any given value of k.
(c) If $k \geqslant 0$ then the equation has (at least) one real solution x.
(d) The equation never has a repeated solution x for any given values of k and n.

2011, Question 1a:

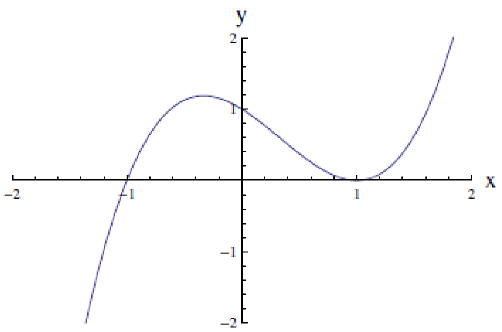
A. A sketch of the graph $y=x^3-x^2-x+1$ appears on which of the following axes?



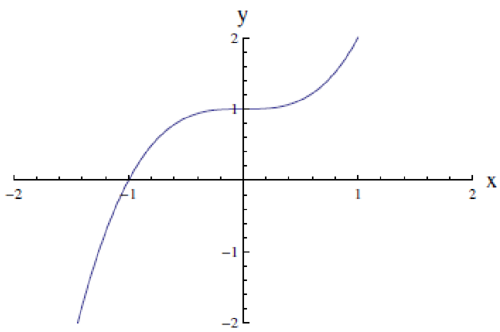
(a)



(b)



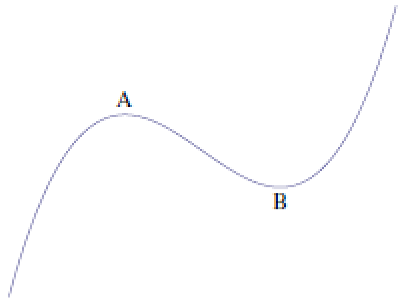
(c)



(d)

2012, Question 3:

Let $f(x) = x^3 + ax^2 + bx + c$, where the coefficients a, b and c are real numbers. The figure below shows a section of the graph of $y = f(x)$. The curve has two distinct turning points; these are located at A and B, as shown. (Note that the axes have been omitted deliberately.)



(i) Find a condition on the coefficients a, b, c such that the curve has two distinct turning points if, and only if, this condition is satisfied.

It may be assumed from now on that the condition on the coefficients in (i) is satisfied.

(ii) Let x_1 and x_2 denote the x coordinates of A and B, respectively. Show that

$$x_2 - x_1 = \frac{2}{3}\sqrt{a^2 - 3b}.$$

(iii) Suppose now that the graph of $y = f(x)$ is translated so that the turning point at A now lies at the origin. Let $g(x)$ be the cubic function such that $y = g(x)$ has the translated graph. Show that

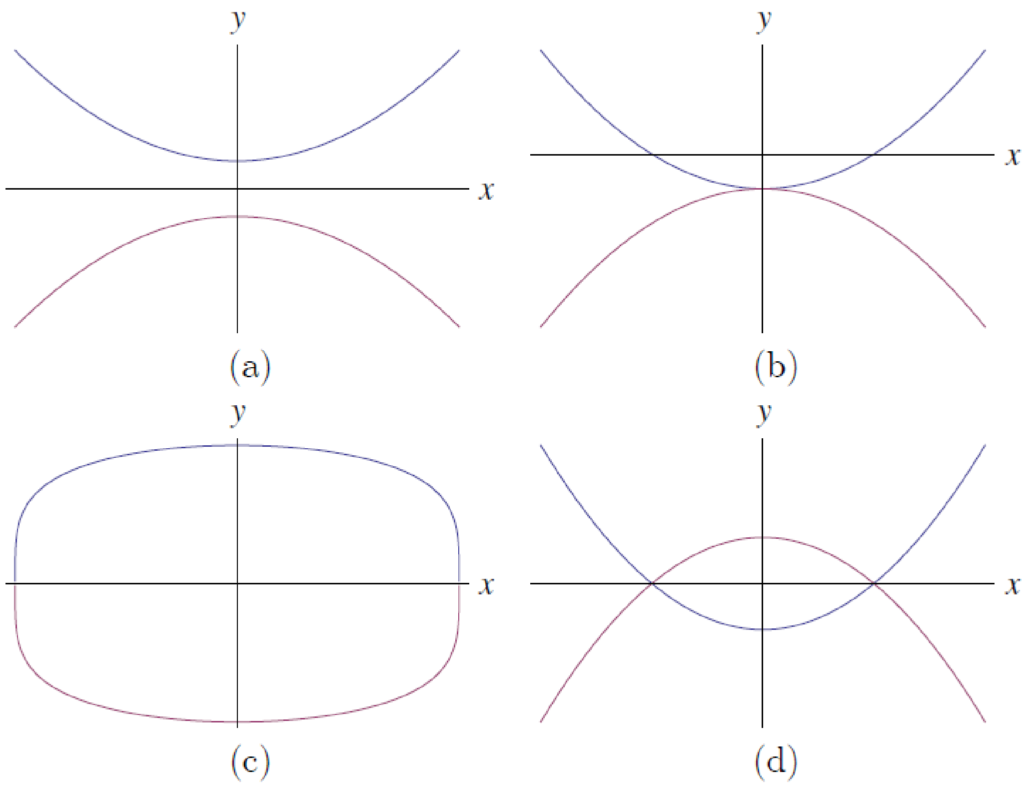
$$g(x) = x^2\left(x - \sqrt{a^2 - 3b}\right).$$

(iv) Let R be the area of the region enclosed by the x-axis and the graph $y = g(x)$. Show that if a and b are rational then R is also rational.

(v) Is it possible for R to be a non-zero rational number when a and b are both irrational? Justify your answer.

2013, Question 1d:

D. Which of the following sketches is a graph of $x^4 - y^2 = 2y + 1$?



2013, Question 1g:

G. Let $n \geqslant 2$ be an integer and $p_n(x)$ be the polynomial

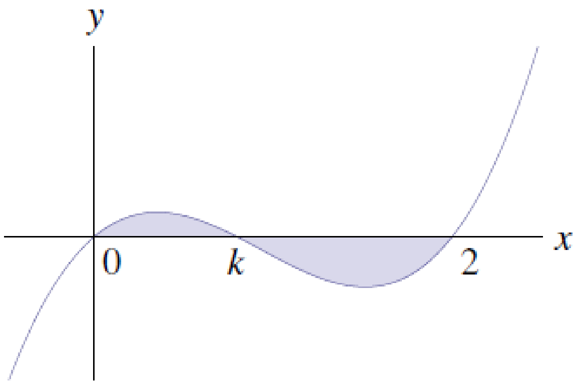
$$p_n(x) = (x-1) + (x-2) + \cdots + (x-n).$$

What is the remainder when $p_n(x)$ is divided by $p_{n-1}(x)$?

(a) $\frac{n}{2}$; (b) $\frac{n+1}{2}$; (c) $\frac{n^2+n}{2}$; (d) $\frac{-n}{2}$.

2013, Question 3:

Let $0 < k < 2$. Below is sketched a graph of $y = f_k(x)$ where $f_k(x) = x(x-k)(x-2)$. Let $A(k)$ denote the area of the shaded region.



(i) Without evaluating them, write down an expression for $A(k)$ in terms of two integrals.

(ii) Explain why $A(k)$ is a polynomial in k of degree 4 or less. [You are not required to calculate $A(k)$ explicitly.]

(iii) Verify that $f_k(1+t) = -f_{2-k}(1-t)$ for any t.

(iv) How can the graph of $y = f_k(x)$ be transformed to the graph of $y = f_{2-k}(x)$?

Deduce that $A(k) = A(2-k)$.

(v) Explain why there are constants a, b, c such that

$$A(k) = a(k-1)^4 + b(k-1)^2 + c.$$

[You are not required to calculate a, b, c explicitly.]

2014, Question 2:

Let a and b be real numbers. Consider the cubic equation

$$x^3 + 2bx^2 - a^2x - b^2 = 0. \qquad (*)$$

(i) Show that if $x = 1$ is a solution of $(*)$ then

$$1 - \sqrt{2} \leqslant b \leqslant 1 + \sqrt{2}.$$

(ii) Show that there is no value of b for which $x = 1$ is a repeated root of $(*)$.

(iii) Given that $x = 1$ is a solution, find the value of b for which $(*)$ has a repeated root.

For this value of b, does the cubic

$$y = x^3 + 2bx^2 - a^2x - b^2$$

have a maximum or minimum at its repeated root?