Core 1 Coordinate Geometry Questions
(From the Oxford MAT Tests)

For answers, see the MAT website

Specimen A, Question 1c:

C. What is the reflection of the point $(3,4)$ in the line $3x+4y=50$?

(a) $(9,12)$ (b) $(6,8)$ (c) $(12,16)$ (d) $(16,12)$

Specimen B, Question 1a:

A. The point lying between $P(2,3)$ and $Q(8,-3)$ which divides the line PQ in the ratio $1:2$ has co-ordinates

(a) $(4,-1)$ (b) $(6,-2)$ (c) $\left(\frac{14}{3},2\right)$ (d) $(4,1)$

Specimen B, Question 4:

Let P and Q be the points with co-ordinates $(7,1)$ and $(11,2)$.

(i) The mirror image of the point P in the x-axis is the point R with co-ordinates $(7,-1)$. Mark the points P, Q and R on the grid provided opposite.

(ii) Consider paths from P to Q each of which consists of two straight line segments PX and XQ where X is a point on the x-axis. Find the length of the shortest such parth, giving clear reasoning for your answer. (You may refer to the diagram to help your explanation, if you wish.)

(iii) Sketch in the line ℓ with equation $y=x$. Find the co-ordinates of S, the mirror image in the line ℓ of the point Q, and mark in the point S.

(iv) Consider paths from P to Q each of which consists of three straight line segments PY, YZ and ZQ, where Y is on the x-axis and Z is on the line ℓ. Find the shortest such path, giving clear reasoning for your answer.

2007, Question 1d:

D. The point on the circle

$$(x-5)^2+(y-4)^2=4$$

which is closest to the circle

$$(x-1)^2+(y-1)^2=1$$

is

(a) $(3.4, 2.8)$, (b) $(3, 4)$, (c) $(5, 2)$, (d) $(3.8, 2.4)$.

2009, Question 1b:

B. The point on the circle

$$x^2+y^2+6x+8y=75,$$

which is closest to the origin, is at what distance from the origin?

(a) 3, (b) 4, (c) 5, (d) 10.

2010, Question 1a:

A. The values of k for which the line $y = kx$ intersects the parabola $y = (x-1)^2$ are precisely

(a) $k \leqslant 0$, (b) $k \geqslant -4$, (c) $k \geqslant 0$ or $k \leqslant -4$, (d) $-4 \leqslant k \leqslant 0$.

2010, Question 2:

Suppose that a, b, c are *integers* such that

$$a\sqrt{2} + b = c\sqrt{3}.$$

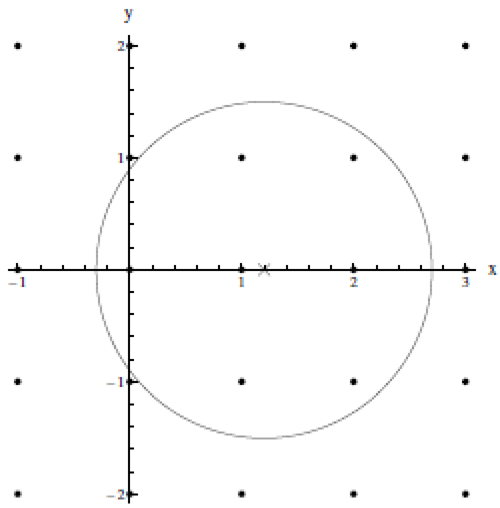
(i) By squaring both sides of the equation, show that $a = b = c = 0$.

[You may assume that $\sqrt{2}, \sqrt{3}$ and $\sqrt{2/3}$ are all irrational numbers. An irrational number is one which cannot be written in the form p/q where p and q are integers.]

(ii) Suppose now that m, n, M, N are *integers* such that the distance from the point (m, n) to $\left(\sqrt{2}, \sqrt{3}\right)$ equals the distance from (M, N) to $\left(\sqrt{2}, \sqrt{3}\right)$.

Show that $m = M$ and $n = N$.

Given real numbers a, b and a positive number r, let $N(a, b, r)$ be the number of integer pairs x, y such that the distance between the points (x, y) and (a, b) is less than or equal to r. For example, we see that $N(1.2, 0, 1.5) = 7$ in the diagram below.



(iii) Explain why $N(0.5, 0.5, r)$ is a multiple of 4 for any value of r.

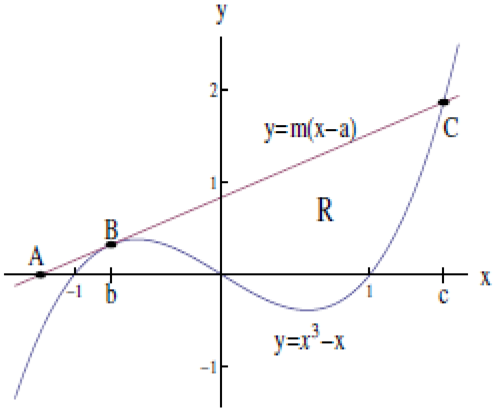
(iv) Let k be any positive integer. Explain why there is a positive number r such that

$$N\left(\sqrt{2}, \sqrt{3}, r\right) = k.$$

2011, Question 3:

The graphs of $y = x^3 - x$ and $y = m(x-a)$ are drawn on the axes below. Here $m > 0$ and $a \leqslant -1$.

The line $y = m(x-a)$ meets the x-axis at $A = (a, 0)$, *touches* the cubic $y = x^3 - x$ at B and intersects again with the cubic at C. The x-coordinates of B and C are respectively b and c.



(i) Use the fact that the line and cubic *touch* when $x = b$, to show that $m = 3b^2 - 1$.

(ii) Show further that

$$a = \frac{2b^3}{3b^2 - 1}.$$

(iii) If $a = -10^6$, what is the approximate value of b?

(iv) Using the fact that

$$x^3 - x - m(x-a) = (x-b)^2(x-c)$$

(which you need not prove), show that $c = -2b$.

(v) R is the finite region bounded above by the line $y = m(x-a)$ and bounded below by the cubic $y = x^3 - x$. For what value of a is the area of R largest?

Show that the largest possible area of R is $\frac{27}{4}$.

2012, Question 1a:

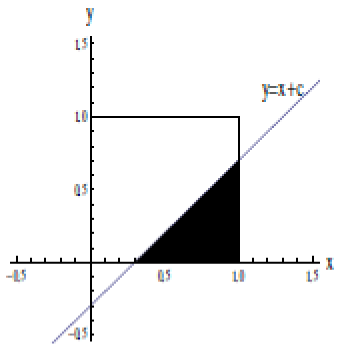
A. Which of the following lines is a tangent to the circle with equation

$$x^2 + y^2 = 4\,?$$

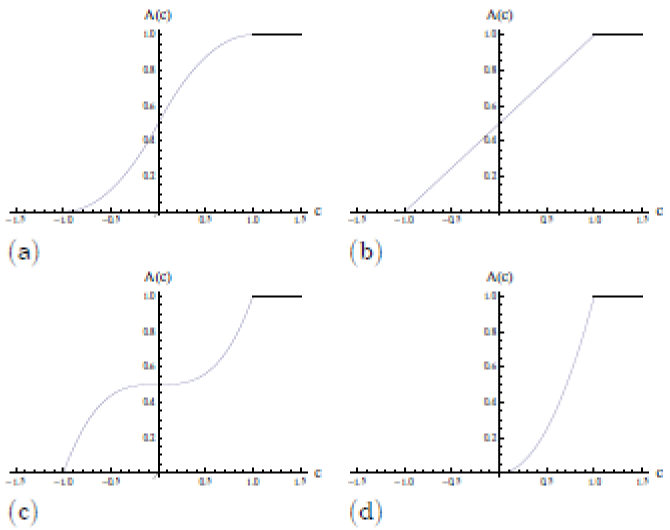
(a) $x + y = 2$; (b) $y = x - 2\sqrt{2}$; (c) $x = \sqrt{2}$; (d) $y = \sqrt{2} - x$.

2012, Question 1d:

D. Shown below is a diagram of the square with vertices $(0,0)$, $(0,1)$, $(1,1)$, $(1,0)$ and the line $y = x + c$. The shaded region is the region of the square which lies below the line; this shaded region has area $A(c)$.



Which of the following graphs shows $A(c)$ as c varies?



2012, Question 1i:

I. The vertices of an equilateral triangle are labelled X, Y and Z. The points X, Y and Z lie on a circle of circumference 10 units. Let P and A be the numerical values of the triangle's perimeter and area, respectively. Which of the following is true?

(a) $\dfrac{A}{P} = \dfrac{5}{4\pi}$; (b) $P < A$; (c) $\dfrac{P}{A} = \dfrac{10}{3\pi}$; (d) P^2 is rational.

2014, Question 1d:

D. The reflection of the point $(1, 0)$ in the line $y = mx$ has coordinates

(a) $\left(\frac{m^2+1}{m^2-1}, \frac{m}{m^2-1}\right)$, (b) $(1, m)$, (c) $(1-m, m)$,

(d) $\left(\frac{1-m^2}{1+m^2}, \frac{2m}{1+m^2}\right)$, (e) $(1-m^2, m)$.