Core 1 Basic Algebra Questions
(From the Oxford MAT Tests)

For answers, see the MAT website

Specimen A, Question 1e:

E. The fact that

$$6 \times 7 = 42,$$

is a counter-example to which of the following statements?

(a) the product of any two odd integers is odd;

(b) if the product of two integers is not a multiple of 4 then the integers are not consecutive;

(c) if the product of two integers is a multiple of 4 then the integers are not consecutive;

(d) any even integer can be written as the product of two even integers.

Specimen A, Question 1g:

G. The inequalities $x^2 + 3x + 2 > 0$ and $x^2 + x < 2$, are met by all x in the region:

(a) $x < -2$;

(b) $-1 < x < 1$;

(c) $x > -1$;

(d) $x > -2$.

Specimen A, Question 3:

In this question we shall consider the function $f(x)$ defined by

$$f(x) = x^2 - 2px + 3$$

where p is a constant.

(i) Show that the function $f(x)$ has one stationary value in the range $0 < x < 1$ if $0 < p < 1$, and no stationary values in that range otherwise.

In the remainder of the question we shall be interested in the smallest value attained by $f(x)$ in the range $0 \leqslant x \leqslant 1$. Of course, this value, which we shall call m, will depend on p.

(ii) Show that if $p \geqslant 1$ then $m = 4 - 2p$.

(iii) What is the value of m if $p \leqslant 0$?

(iv) Obtain a formula for m in terms of p, valid for $0 < p < 1$.

(v)Using the axes opposite, sketch the graph of m as a function of p in the range $-2 \leqslant p \leqslant 2$.

Specimen B, Question 1d:

D. The numbers x and y satisfy the following inequalities

$$\begin{aligned} 2x + 3y &\leqslant 23, \\ x + 2 &\leqslant 3y, \\ 3y + 1 &\leqslant 4x. \end{aligned}$$

The largest possible value of x is

(a) 6 (b) 7 (c) 8 (d) .9

Specimen B, Question 1f:

F. The turning point of the parabola

$$y = x^2 - 2ax + 1$$

is closest to the origin when

(a) $a = 0$ (b) $a = \pm 1$ (c) $a = \pm\frac{1}{\sqrt{2}}$ or $a = 0$ (d) $a = \pm\frac{1}{\sqrt{2}}$.

Specimen B, Question 1g:

G. The four digit number 2652 is such that any two consecutive digits from it make a multiple of 13. Another number N has this same property, is 100 digits long, and begins in a 9. What is the last digit of N?

(a) 2 (b) 3 (c) 6 (d) 9

Specimen B, Question 1j:

J. Into how many regions is the plane divided when the following three parabolas are drawn?

$$\begin{aligned} y &= x^2 \\ y &= x^2 - 2x \\ y &= x^2 + 2x + 2. \end{aligned}$$

(a) 4 (b) 5 (c) 6 (d) 7

2009, Question 1c:

C. Given a real constant c, the equation

$$x^4 = (x - c)^2$$

has four real solutions (including possible repeated roots) for

(a) $c \leqslant \frac{1}{4}$, (b) $-\frac{1}{4} \leqslant c \leqslant \frac{1}{4}$, (c) $c \leqslant -\frac{1}{4}$, (d) all values of c.

2011, Question 1b:

B. A rectangle has perimeter P and area A. The values P and A must satisfy

(a) $P^3 > A$, (b) $A^2 > 2P + 1$, (c) $P^2 \geqslant 16A$, (d) $PA \geqslant A + P$.

2011, Question 2:

Suppose that x satisfies the equation

$$x^3 = 2x + 1. \qquad (*)$$

(i) Show that

$$x^4 = x + 2x^2 \qquad \text{and} \qquad x^5 = 2 + 4x + x^2.$$

(ii) For every integer $k \geqslant 0$, we can uniquely write

$$x^k = A_k + B_k x + C_k x^2$$

where A_k, B_k, C_k are integers. So, in part (i), it was shown that

$$A_4 = 0,\ B_4 = 1, C_4 = 2 \qquad \text{and} \qquad A_5 = 2,\ B_5 = 4,\ C_5 = 1.$$

Show that

$$A_{k+1} = C_k, \qquad B_{k+1} = A_k + 2C_k, \qquad C_{k+1} = B_k.$$

(iii) Let

$$D_k = A_k + C_k - B_k.$$

Show that $D_{k+1} = -D_k$ and hence that

$$A_k + C_k = B_k + (-1)^k.$$

(iv) Let $F_k = A_{k+1} + C_{k+1}$. Show that

$$F_k + F_{k+1} = F_{k+2}.$$

2012, Question 1g:

G. There are *positive* real numbers x and y which solve the equations

$$2x + ky = 4, \qquad x + y = k$$

for

(a) all values of k; (b) no values of k; (c) $k = 2$ only; (d) only $k > -2$.

2013, Question 1a:

A. For what values of the real number a does the quadratic equation

$$x^2 + ax + a = 1$$

have distinct real roots?

(a) $a \neq 2$; (b) $a > 2$; (c) $a = 2$; (d) all values of a.

2014, Question 1a:

A. The inequality

$$x^4 < 8x^2 + 9$$

is satisfied precisely when

(a) $-3 < x < 3$; (b) $0 < x < 4$; (c) $1 < x < 3$; (d) $-1 < x < 9$; (e) $-3 < x < -1$.
