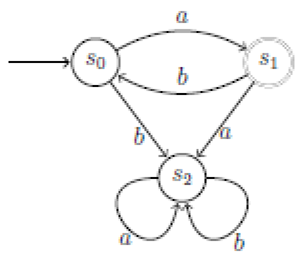
A *finite automaton* is a mathematical model of a simple computing device. A small finite automaton is illustrated below.



A finite automaton has some *finite* number of states; the above automaton has three states, labelled s_0, s_1 and s_2. The initial state, s_0, is indicated with an incoming arrow. The automaton receives *inputs* (e.g. via button presses), which might cause it to change state. In the example, the inputs are a and b. The state changes are illustrated by arrows; for example, if the automaton is in state s_1 and it receives input b, it changes to state s_0; if it is in state s_2 and receives either input, it remains in state s_2. (For each state, there is precisely one out-going arrow for each input.)

Some of the states are defined to be *accepting states*; in the example, just s_1 is defined to be an accepting state, represented by the double circle. A *word* is a sequence of inputs. The automaton *accepts* a word w if that sequence of inputs leads to an accepting state from the initial state. For example, the above automaton accepts the word aba.

(i) Write down a description of the set of words accepted by the above automaton. A clear but informal description will suffice.

(ii) Suppose we alter the above automaton by swapping accepting and non-accepting states; i.e. we make s_0 and s_2 accepting, and make s_1 non-accepting. Write down a description of the set of words accepted by this new automaton. Again, a clear but informal description will suffice.

(iii) Draw an automaton that accepts all words containing an even number (possibly zero) of a's and any number of b's (and no other words).

(iv) Now draw an automaton that accepts all words containing an even number of a's *or* an odd number of b's (and no other words).

Let a^n represent n consecutive a's. Let L be the set of all words of the form $a^n b^n$ where $n = 0, 1, 2, \ldots$; i.e. all words composed of some number of a's followed by the *same* number of b's. We will show that no finite automaton accepts precisely this set of words.