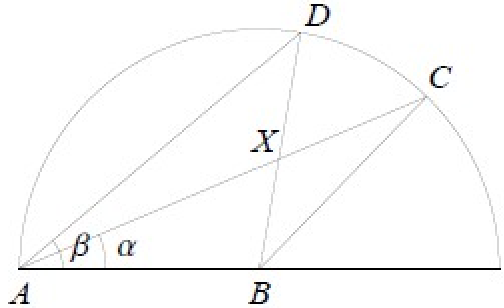
In the diagram below is sketched a semicircle with centre B and radius 1. Three points A, C, D lie on the semicircle as shown with α denoting angle CAB and β denoting angle DAB. The triangles ABC and ABD intersect in a triangle ABX.

Throughout the question we shall consider the value of α fixed. Assume for now that $0 < \alpha \leqslant \beta \leqslant \pi/2$.



(i) Show that the area of the triangle ABC equals

$$\frac{1}{2}\sin(2\alpha).$$

(ii) Let

$$F = \frac{\text{area of triangle } ABX}{\text{area of triangle } ABC}.$$

Without calculation, explain why, for every k in the range $0 \leqslant k \leqslant 1$, there is a unique value of β such that $F = k$.

(iii) Find the value of β such that $F = 1/2$.

(iv) Show that

$$F = \frac{\sin(2\beta)\sin\alpha}{\sin(2\beta - \alpha)\sin(2\alpha)}.$$

(v) Suppose now that $0 < \beta < \alpha \leqslant \pi/2$. Write down, without further calculation, an expression for the area of ABX and hence a formula for F.