

Let a and b be real numbers. Consider the cubic equation

$$x^3 + 2bx^2 - a^2x - b^2 = 0. \quad (*)$$

(i) Show that if $x = 1$ is a solution of $(*)$ then

$$1 - \sqrt{2} \leq b \leq 1 + \sqrt{2}.$$

(ii) Show that there is no value of b for which $x = 1$ is a repeated root of $(*)$.

(iii) Given that $x = 1$ is a solution, find the value of b for which $(*)$ has a repeated root.

For this value of b , does the cubic

$$y = x^3 + 2bx^2 - a^2x - b^2$$

have a maximum or minimum at its repeated root?