

AB-words are “words” formed from the letters **A** and **B** according to certain rules. The rules are applied starting with the empty word, containing no letters. The basic rules are:

(1) If the current word is x , then it can be replaced with the word that starts with **A**, followed by x and ending with **B**, written AxB .

(2) If the current word ends with **B**, the final **B** can be removed.

(i) Show how the word **AAAB** can be produced.

(ii) Describe precisely all the words that can be produced with these two rules. Justify your answer. You might like to write A^i for the word containing just i consecutive copies of **A**, and similarly for **B**; for example $A^3B^2 = AAABB$.

We now add a third rule:

(3) Reverse the word, and replace every **A** by **B**, and every **B** by **A**.

For example, applying this rule to **AAAB** would give **ABBB**.

(iii) Describe precisely all the words that can be produced with these three rules. Justify your answer.

Finally, we add a fourth rule:

(4) Reverse the word.

(iv) Show that every word consisting of **A**s and **B**s can be formed using these four rules. Hint: show how, if we have produced the word w , we can produce (a) the word Aw , and (b) the word Bw ; hence deduce the result.