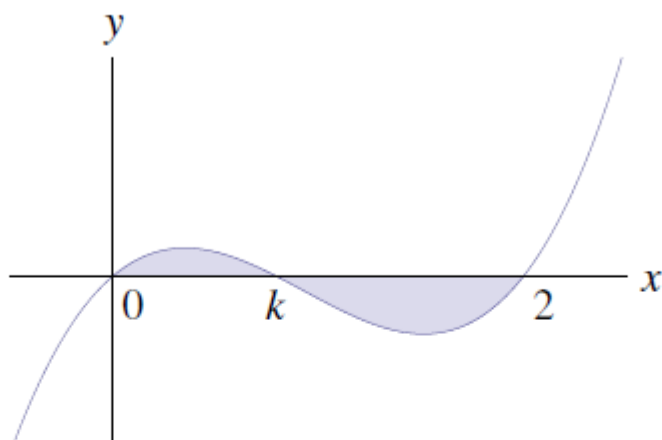


Let $0 < k < 2$. Below is sketched a graph of $y = f_k(x)$ where $f_k(x) = x(x - k)(x - 2)$. Let $A(k)$ denote the area of the shaded region.



(i) Without evaluating them, write down an expression for $A(k)$ in terms of two integrals.

(ii) Explain why $A(k)$ is a polynomial in k of degree 4 or less. [You are not required to calculate $A(k)$ explicitly.]

(iii) Verify that $f_k(1 + t) = -f_{2-k}(1 - t)$ for any t .

(iv) How can the graph of $y = f_k(x)$ be transformed to the graph of $y = f_{2-k}(x)$?

Deduce that $A(k) = A(2 - k)$.

(v) Explain why there are constants a, b, c such that

$$A(k) = a(k - 1)^4 + b(k - 1)^2 + c.$$

[You are not required to calculate a, b, c explicitly.]