

Let

$$f(x) = x + 1 \quad \text{and} \quad g(x) = 2x.$$

We will, for example, write fg to denote the function "perform g then perform f " so that

$$fg(x) = f(g(x)) = 2x + 1.$$

If $i \geq 0$ is an integer we will, for example, write f^i to denote the function which performs f i times, so that

$$f^i(x) = \underbrace{fff \cdots f}_{i \text{ times}}(x) = x + i.$$

(i) Show that

$$f^2g(x) = gf(x).$$

(ii) Note that

$$gf^2g(x) = 4x + 4.$$

Find all the other ways of combining f and g that result in the function $4x + 4$.

(iii) Let $i, j, k \geq 0$ be integers. Determine the function

$$f^i g f^j g f^k(x).$$

(iv) Let $m \geq 0$ be an integer. How many different ways of combining the functions f and g are there that result in the function $4x + 4m$?