

An $n \times n$ grid consists of squares arranged in n rows and n columns; for example, a chessboard is an 8×8 grid. Let us call a *semi-grid* of size n the lower left part of an $n \times n$ grid – that is, the squares located on or below the grid's diagonal. For example, Figure C shows an example of a semi-grid of size 4.

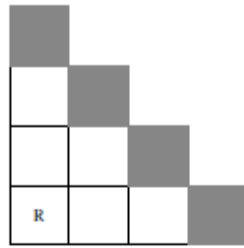


Figure C

Let us suppose that a robot is located in the lower-left corner of the grid. The robot can move only up or right, and its goal is to reach one of the *goal squares*, which are all located on the semi-grid's diagonal. In the example shown in Figure C, the robot is initially located in the square denoted with R, and the goal squares are shown in grey. Let us call a *solution* a sequence of the robot's moves that leads the robot from the initial location to some goal square.

- (i) Write down all 8 solutions for a robot on a semi-grid of size 4.
- (ii) Devise a concise way of representing the possible journeys of the robot in a semi-grid of size n . In your notation, which of the journeys are solutions?
- (iii) Write down a formula for the number of possible solutions in a semi-grid of size n . Explain why your formula is correct.

Now let us change the problem slightly and redefine a goal square as any square that can be described as follows:

- the lower-left square is not a goal square;
- each square that is located immediately above or immediately to the right of a non-goal square is a goal square; and
- each square that is located immediately above or immediately to the right of a goal square is a non-goal square.

Furthermore, let us assume that, upon reaching a goal square, the robot may decide to stop or to continue moving (provided that there are more allowed moves).

- (iv) With these modifications in place, write down all the solutions in a semi-grid of size 4, and all the solutions in a semi-grid of size 5.
- (v) How many solutions are there now in a semi-grid of size n , where n is a positive integer? You may wish to consider separately the cases where n is even or odd.