Suppose that a, b, c are *integers* such that

$$a\sqrt{2} + b = c\sqrt{3}.$$

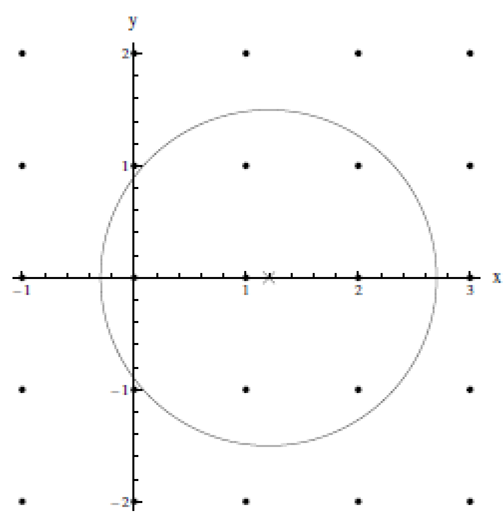
(i) By squaring both sides of the equation, show that $a = b = c = 0$.

[You may assume that $\sqrt{2}, \sqrt{3}$ and $\sqrt{2/3}$ are all irrational numbers. An irrational number is one which cannot be written in the form p/q where p and q are integers.]

(ii) Suppose now that m, n, M, N are *integers* such that the distance from the point (m, n) to $(\sqrt{2}, \sqrt{3})$ equals the distance from (M, N) to $(\sqrt{2}, \sqrt{3})$.

Show that $m = M$ and $n = N$.

Given real numbers a, b and a positive number r, let $N(a, b, r)$ be the number of integer pairs x, y such that the distance between the points (x, y) and (a, b) is less than or equal to r. For example, we see that $N(1.2, 0, 1.5) = 7$ in the diagram below.



(iii) Explain why $N(0.5, 0.5, r)$ is a multiple of 4 for any value of r.

(iv) Let k be any positive integer. Explain why there is a positive number r such that

$$N\left(\sqrt{2}, \sqrt{3}, r\right) = k.$$