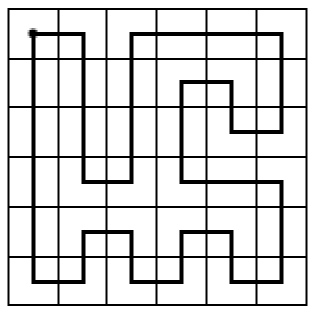
Given an $n \times n$ grid of squares, where $n > 1$, a *tour* is a path drawn within the grid such that:

- along its way the path moves, horizontally or vertically, from the centre of one square to the centre of an adjacent square;
- the path starts and finishes in the same square;
- the path visits the centre of every other square just once.

For example, below is a tour drawn in a 6×6 grid of squares which starts and finishes in the top-left square.



For parts (i)-(iv) it is assumed that n is *even*.

(i) With the aid of a diagram, show how a tour, which starts and finishes in the top-left square, can be drawn in any $n \times n$ grid.

(ii) Is a tour still possible if the start/finish point is changed to the centre of a different square? Justify your answer.

Suppose now that a robot is programmed to move along a tour of an $n \times n$ grid. The robot understands two commands:

- command R which turns the robot clockwise through a right angle;
- command F which moves the robot forward to the centre of the next square.

The robot has a program, a list of commands, which it performs in the given order to complete a tour; say that, in total, command R appears r times in the program and command F appears f times.

(iii) Initially the robot is in the top-left square pointing to the right. Assuming the first command is an F, what is the value of f? Explain also why $r + 1$ is a multiple of 4.

(iv) Must the results of part (iii) still hold if the robot starts and finishes at the centre of a different square? Explain your reasoning.

(v) Show that a tour of an $n \times n$ grid is not possible when n is odd.