

For a positive whole number n , the function $f_n(x)$ is defined by

$$f_n(x) = (x^{2n-1} - 1)^2.$$

(i) On the axes provided opposite, sketch the graph of $y = f_2(x)$ labelling where the graph meets the axes.

(ii) On the same axes sketch the graph of $y = f_n(x)$ where n is a large positive integer.

(iii) Determine

$$\int_0^1 f_n(x) \, dx.$$

(iv) The *positive* constants A and B are such that

$$\int_0^1 f_n(x) \, dx \leq 1 - \frac{A}{n+B} \quad \text{for all } n \geq 1.$$

Show that

$$(3n-1)(n+B) \geq A(4n-1)n,$$

and explain why $A \leq 3/4$.

(v) When $A = 3/4$, what is the smallest possible value of B ?