

A list of real numbers x_1, x_2, x_3, \dots is defined by $x_1 = 1, x_2 = 3$ and then for $n \geq 3$ by

$$x_n = 2x_{n-1} - x_{n-2} + 1.$$

So, for example,

$$x_3 = 2x_2 - x_1 + 1 = 2 \times 3 - 1 + 1 = 6.$$

(i) Find the values of x_4 and x_5 .

(ii) Find values of real constants A, B, C such that for $n = 1, 2, 3,$

$$x_n = A + Bn + Cn^2. \quad (*)$$

(iii) Assuming that equation $(*)$ holds true for all $n \geq 1,$ find the smallest n such that $x_n \geq 800$.

(iv) A second list of real numbers y_1, y_2, y_3, \dots is defined by $y_1 = 1$ and

$$y_n = y_{n-1} + 2n$$

Find, explaining your reasoning, a formula for y_n which holds for $n \geq 2$.

What is the approximate value of x_n/y_n for large values of n ?