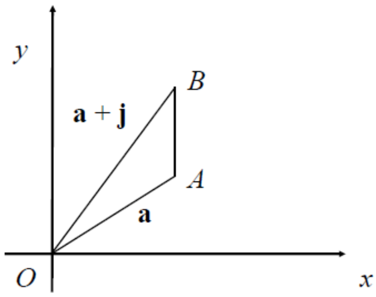
Core 4 Vectors Questions
(From AEA Papers)

For answers, see the AEA website

2003, Question 1:

1. **Figure 1**



The point A is a distance 1 unit from the fixed origin O. Its position vector is $\mathbf{a} = \frac{1}{\sqrt{2}}(\mathbf{i} + \mathbf{j})$.

The point B has position vector $\mathbf{a} + \mathbf{j}$, as shown in Figure 1.

By considering ΔOAB, prove that $\tan \frac{3\pi}{8} = 1 + \sqrt{2}$.

(5)

2005, Question 5:

5. The point A has position vector $7\mathbf{i} + 2\mathbf{j} - 7\mathbf{k}$ and the point B has position vector $12\mathbf{i} + 3\mathbf{j} - 15\mathbf{k}$.

(*a*) Find a vector for the line L_1 which passes through A and B. **(2)**

The line L_2 has vector equation

$$\mathbf{r} = -4\mathbf{i} + 12\mathbf{k} + \mu(\mathbf{i} - 3\mathbf{k}).$$

(*b*) Show that L_2 passes through the origin O. **(1)**

(*c*) Show that L_1 and L_2 intersect at a point C and find the position vector of C. **(3)**

(*d*) Find the cosine of $\angle OCA$. **(3)**

(*e*) Hence, or otherwise, find the shortest distance from O to L_1. **(3)**

(*f*) Show that $|\overrightarrow{CO}| = |\overrightarrow{AB}|$. **(2)**

(*g*) Find a vector equation for the line which bisects $\angle OCA$. **(5)**

2006, Question 5:

5. The lines L_1 and L_2 have vector equations

L_1: $\mathbf{r} = -2\mathbf{i} + 11.5\mathbf{j} + \lambda(3\mathbf{i} - 4\mathbf{j} - \mathbf{k})$,

L_2: $\mathbf{r} = 11.5\mathbf{i} + 3\mathbf{j} + 8.5\mathbf{k} + \mu(7\mathbf{i} + 8\mathbf{j} - 11\mathbf{k})$,

where λ and μ are parameters.

(*a*) Show that L_1 and L_2 do not intersect. **(5)**

(*b*) Show that the vector $(2\mathbf{i} + \mathbf{j} + 2\mathbf{k})$ is perpendicular to both L_1 and L_2. **(2)**

The point A lies on L_1, the point B lies on L_2 and AB is perpendicular to both L_1 and L_2.

(*c*) Find the position vector of the point A and the position vector of the point B. **(8)**

2007, Question 7:

7. The points O, P and Q lie on a circle C with diameter OQ. The position vectors of P and Q, relative to O, are $\mathbf{p}$ and $\mathbf{q}$ respectively.

(a) Prove that $\mathbf{p}.\mathbf{q} = |\mathbf{p}|^2$.

(3)

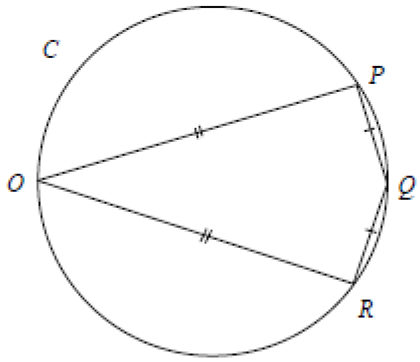


Figure 3

The point R also lies on C and $OPQR$ is a kite K as shown in Figure 3. The point S has position vector, relative to O, of $\lambda\mathbf{q}$, where λ is a constant. Given that $\mathbf{p} = \mathbf{i} + 2\mathbf{j} - \mathbf{k}$, $\mathbf{q} = 2\mathbf{i} + \mathbf{j} - 2\mathbf{k}$ and that OQ is perpendicular to PS, find

(b) the value of λ,

(2)

(c) the position vector of R,

(3)

(d) the area of K.

(4)

Another circle C_1 is drawn inside K so that the 4 sides of the kite are each tangents to C_1.

(e) Find the radius of C_1 giving your answer in the form $(\sqrt{2} - 1)\sqrt{n}$, where n is an integer.

(5)

A second kite K_1 is similar to K and is drawn inside C_1.

(f) Find that area of K_1.

(3)

2008, Question 7:

7. Relative to a fixed origin O, the position vectors of the points A, B and C are

$$\overrightarrow{OA} = -3\mathbf{i} + \mathbf{j} - 9\mathbf{k}, \quad \overrightarrow{OB} = \mathbf{i} - \mathbf{k}, \quad \overrightarrow{OC} = 5\mathbf{i} + 2\mathbf{j} - 5\mathbf{k} \text{ respectively.}$$

(a) Find the cosine of angle ABC.

(4)

The line L is the angle bisector of angle ABC.

(b) Show that an equation of L is $\mathbf{r} = \mathbf{i} - \mathbf{k} + t(\mathbf{i} + 2\mathbf{j} - 7\mathbf{k})$.

(4)

(c) Show that $|\overrightarrow{AB}| = |\overrightarrow{AC}|$.

(2)

The circle S lies inside triangle ABC and each side of the triangle is a tangent to S.

(d) Find the position vector of the centre of S.

(7)

(e) Find the radius of S.

(5)

2009, Question 7:

7. Relative to a fixed origin O the points A, B and C have position vectors

$$\mathbf{a} = -\mathbf{i} + \tfrac{4}{3}\mathbf{j} + 7\mathbf{k}, \quad \mathbf{b} = 4\mathbf{i} + \tfrac{4}{3}\mathbf{j} + 2\mathbf{k} \text{ and } \mathbf{c} = 6\mathbf{i} + \tfrac{16}{3}\mathbf{j} + 2\mathbf{k} \text{ respectively.}$$

(a) Find the cosine of angle ABC.

(3)

The quadrilateral $ABCD$ is a kite K.

(b) Find the area of K.

(3)

A circle is drawn inside K so that it touches each of the 4 sides of K.

(c) Find the radius of the circle, giving your answer in the form $p\sqrt{(q)} - q\sqrt{(p)}$, where p and q are positive integers.

(5)

(d) Find the position vector of the point D.

(7)

2010, Question 4:

4.

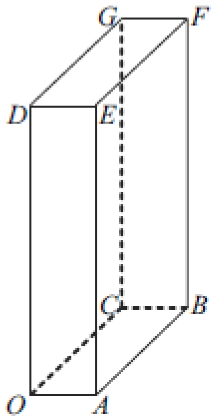


Figure 1

Figure 1 shows a cuboid $OABCDEFG$, where O is the origin, A has position vector $5\mathbf{i}$, C has position vector $10\mathbf{j}$ and D has position vector $20\mathbf{k}$.

(a) Find the cosine of angle CAF.

(4)

Given that the point X lies on AC and that FX is perpendicular to AC,

(b) find the position vector of point X and the distance FX.

(7)

The line l_1 passes through O and through the midpoint of the face $ABFE$. The line l_2 passes through A and through the midpoint of the edge FG.

(c) Show that l_1 and l_2 intersect and find the coordinates of the point of intersection.

(5)

2011, Question 6:

6. The line L has equation

$$\mathbf{r} = \begin{pmatrix} 13 \\ -3 \\ -8 \end{pmatrix} + t\begin{pmatrix} -5 \\ 3 \\ 4 \end{pmatrix}$$

The point P has position vector $\begin{pmatrix} -7 \\ 2 \\ 7 \end{pmatrix}$.

The point P' is the reflection of P in L.

(a) Find the position vector of P'. **(6)**

(b) Show that the point A with position vector $\begin{pmatrix} -7 \\ 9 \\ 8 \end{pmatrix}$ lies on L. **(1)**

(c) Show that angle $PAP' = 120°$. **(3)**

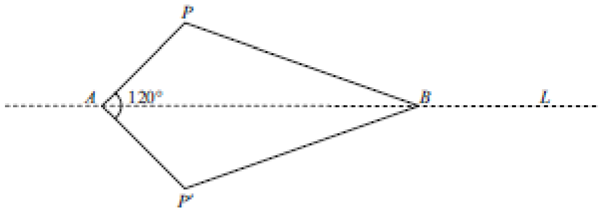


Figure 3

The point B lies on L and $APBP'$ forms a kite as shown in Figure 3.

The area of the kite is $50\sqrt{3}$

(d) Find the position vector of the point B. **(5)**

(e) Show that angle $BPA = 90°$. **(2)**

The circle C passes through the points A, P, P' and B.

(f) Find the position vector of the centre of C. **(2)**

2012, Question 4:

4.

$$\mathbf{a} = \begin{pmatrix} -3 \\ 1 \\ 4 \end{pmatrix}, \quad \mathbf{b} = \begin{pmatrix} 5 \\ -2 \\ 9 \end{pmatrix}, \quad \mathbf{c} = \begin{pmatrix} 8 \\ -4 \\ 3 \end{pmatrix}$$

The points A, B and C with position vectors $\mathbf{a}$, $\mathbf{b}$ and $\mathbf{c}$, respectively, are 3 vertices of a cube.

(a) Find the volume of the cube.

(5)

The points P, Q and R are vertices of a second cube with $\overrightarrow{PQ} = \begin{pmatrix} 3 \\ 4 \\ \alpha \end{pmatrix}$, $\overrightarrow{PR} = \begin{pmatrix} 7 \\ 1 \\ 0 \end{pmatrix}$

and α a positive constant.

(b) Given that angle $QPR = 60°$, find the value of α.

(3)

(c) Find the length of a diagonal of the second cube.

(3)

2014, Question 6:

6. (i) A curve with equation $y = \mathrm{f}(x)$ has $\mathrm{f}(x) \geqslant 0$ for $x \geqslant a$ and

$$A = \int_a^b \mathrm{f}(x)\,\mathrm{d}x \quad \text{and} \quad V = \pi\int_a^b [\mathrm{f}(x)]^2\,\mathrm{d}x$$

where a and b are constants with $b > a$.

Use integration by substitution to show that for the positive constants r and h

$$\pi\int_{a+h}^{b+h} [r + \mathrm{f}(x-h)]^2\,\mathrm{d}x = \pi r^2(b-a) + 2\pi r A + V$$

(3)

(ii)

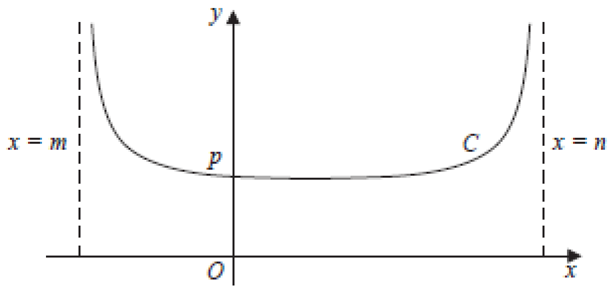


Figure 1

Figure 1 shows part of the curve C with equation $y = 4 + \dfrac{2}{\sqrt{3}\cos x + \sin x}$

This curve has asymptotes $x = m$ and $x = n$ and crosses the y-axis at $(0, p)$.

(a) Find the value of p, the value of m and the value of n.

(4)

(b) Show that the equation of C can be written in the form $y = r + \mathrm{f}(x - h)$ and specify the function f and the constants r and h.

(4)

The region bounded by C, the x-axis and the lines $x = \dfrac{\pi}{6}$ and $x = \dfrac{\pi}{3}$ is rotated through 2π radians about the x-axis.

(c) Find the volume of the solid formed.

(9)