Core 4 Parametric Equation Questions
(From AEA Papers)

For answers, see the AEA website

2002, Question 3:

3. The curve C has parametric equations

$$x = 15t - t^3,\ y = 3 - 2t^2.$$

Find the values of t at the points where the normal to C at (14, 1) cuts C again.

(11)

2003, Question 3:

3. **Figure 2**

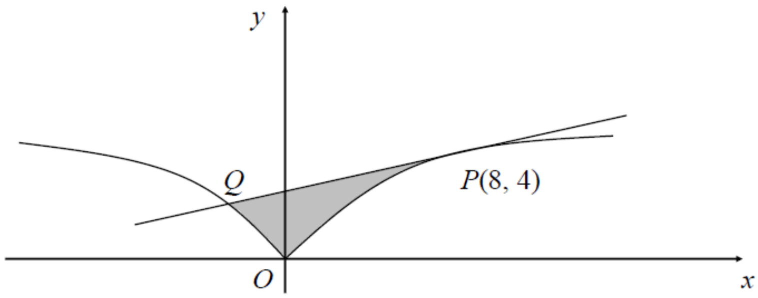

Figure 2 shows a sketch of a part of the curve C with parametric equations

$$x = t^3,\ y = t^2.$$

The tangent at the point $P(8, 4)$ cuts C at the point Q.

Find the area of the shaded region between PQ and C.

(11)

2004, Question 5:

5. (*a*) Given that $y = \ln [t + \sqrt{(1+t^2)}]$, show that $\dfrac{dy}{dt} = \dfrac{1}{\sqrt{(1+t^2)}}$.

(3)

The curve C has parametric equations

$$x = \frac{1}{\sqrt{(1+t^2)}}, \quad y = \ln [t + \sqrt{(1+t^2)}], \quad t \in \mathbb{R}.$$

A student was asked to prove that, for $t > 0$, the gradient of the tangent to C is negative.

The attempted proof was as follows:

$$\begin{aligned} y &= \ln \left(t + \frac{1}{x}\right) \\ &= \ln \left(\frac{tx+1}{x}\right) \\ &= \ln (tx+1) - \ln x \\ \therefore \frac{dy}{dx} &= \frac{t}{tx+1} - \frac{1}{x} \\ &= \frac{\frac{t}{x}}{t + \frac{1}{x}} - \frac{1}{x} \\ &= \frac{t\sqrt{(1+t^2)}}{t+\sqrt{(1+t^2)}} - \sqrt{(1+t^2)} \\ &= -\frac{(1+t^2)}{t+\sqrt{(1+t^2)}} \end{aligned}$$

As $(1+t^2) > 0$, and $t + \sqrt{(1+t^2)} > 0$ for $t > 0$, $\dfrac{dy}{dx} < 0$ for $t > 0$.

(*b*) (i) Identify the error in this attempt.

(ii) Give a correct version of the proof.

(6)

(*c*) Prove that $\ln [-t + \sqrt{(1+t^2)}] = -\ln [t + \sqrt{(1+t^2)}]$.

(3)

(*d*) Deduce that C is symmetric about the x-axis and sketch the graph of C.

(3)

2009, Question 6:

6.

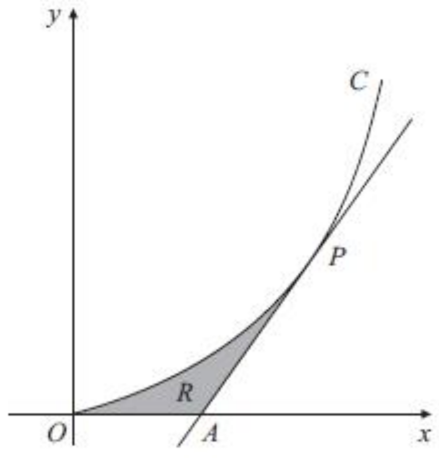

Figure 2

Figure 2 shows a sketch of the curve C with parametric equations

$$x = 2\sin t, \qquad y = \ln(\sec t), \qquad 0 \leqslant t < \frac{\pi}{2}.$$

The tangent to C at the point P, where $t = \frac{\pi}{3}$, cuts the x-axis at A.

(a) Show that the x-coordinate of A is $\frac{\sqrt{3}}{3}(3 - \ln 2)$.

(6)

The shaded region R lies between C, the positive x-axis and the tangent AP as shown in Figure 2.

(b) Show that the area of R is $\sqrt{3}(1 + \ln 2) - 2\ln(2 + \sqrt{3}) - \frac{\sqrt{3}}{6}(\ln 2)^2$.

(11)

2011, Question 4:

4. The curve C has parametric equations

$$x = \cos^2 t$$

$$y = \cos t \sin t$$

where $0 \leqslant t < \pi$

(a) Show that C is a circle and find its centre and its radius.

(5)

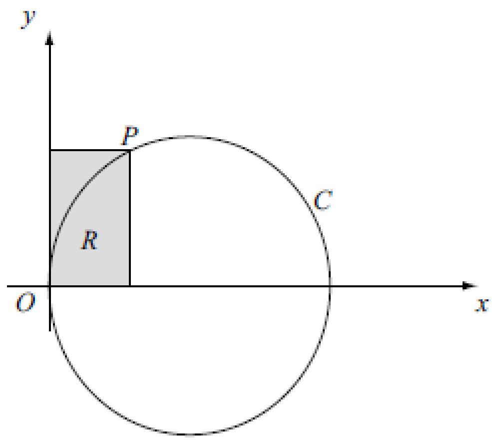

Figure 1

Figure 1 shows a sketch of C. The point P, with coordinates $\left(\cos^2\alpha,\ \cos\alpha \sin\alpha\right)$, $0 < \alpha < \frac{\pi}{2}$, lies on C. The rectangle R has one side on the x-axis, one side on the y-axis and OP as a diagonal, where O is the origin.

(b) Show that the area of R is $\sin\alpha \cos^3\alpha$

(1)

(c) Find the maximum area of R, as α varies.

(7)