Core 4 Differentiation Questions
(From AEA Papers)

For answers, see the AEA website

2002, Question 4:

4. Find the coordinates of the stationary points of the curve with equation

$$x^3 + y^3 - 3xy = 48$$

and determine their nature.

(14)

2007, Question 4:

4. The function h(x) has domain $\mathbb{R}$ and range $\mathrm{h}(x) > 0$, and satisfies

$$\sqrt{\int \mathrm{h}(x)\ \mathrm{d}x} = \int \sqrt{\mathrm{h}(x)}\ \mathrm{d}x.$$

(*a*) By substituting $\mathrm{h}(x) = \left(\frac{\mathrm{d}y}{\mathrm{d}x}\right)^2$, show that

$$\frac{\mathrm{d}y}{\mathrm{d}x} = 2(y + c),$$

where c is constant.

(5)

(*b*) Hence find a general expression for y in terms of x.

(4)

(*c*) Given that $\mathrm{h}(0) = 1$, find $\mathrm{h}(x)$.

(2)

2008, Question 2:

2. The points (x, y) on the curve C satisfy

$$(x+1)(x+2)\frac{dy}{dx} = xy.$$

The line with equation $y = 2x + 5$ is the tangent to C at a point P.

(a) Find the coordinates of P.

(4)

(b) Find the equation of C, giving your answer in the form $y = f(x)$.

(8)

2010, Question 3:

3. The curve C has equation

$$x^2 + y^2 + fxy = g^2,$$

where f and g are constants and $g \neq 0$.

(a) Find an expression in terms of α, β and f for the gradient of C at the point (α, β).

(4)

Given that $f < 2$ and $f \neq -2$ and that the gradient of C at the point (α, β) is 1,

(b) show that $\alpha = -\beta = \dfrac{\pm g}{\sqrt{(2-f)}}$.

(4)

Given that $f = -2$,

(c) sketch C.

(3)

2008, Question 4:

4. **Figure 1**

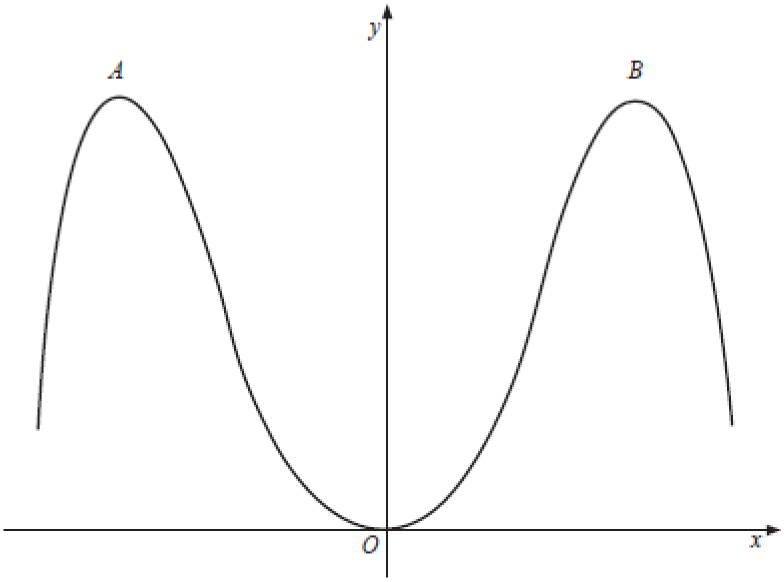

Figure 1 shows a sketch of the curve C with equation

$$y = \cos x \ln(\sec x), \qquad -\frac{\pi}{2} < x < \frac{\pi}{2}.$$

The points A and B are maximum points on C.

(a) Find the coordinates of B in terms of e.

(5)

The finite region R lies between C and the line AB.

(b) Show that the area of R is

$$\frac{2}{e} \arccos\left(\frac{1}{e}\right) + 2\ln\left(e + \sqrt{(e^2 - 1)}\right) - \frac{4}{e}\sqrt{(e^2 - 1)}.$$

[arccos$x$ is an alternative notation for $\cos^{-1}x$]

(8)