Core 3 Trigonometry Questions
(From AEA Papers)

For answers, see the AEA website

2004, Question 7:

7. Triangle ABC, with $BC = a$, $AC = b$ and $AB = c$ is inscribed in a circle. Given that AB is a diameter of the circle and that a^2, b^2 and c^2 are three consecutive terms of an arithmetic progression (arithmetic series),

(*a*) express b and c in terms of a,

(4)

(*b*) verify that $\cot A$, $\cot B$ and $\cot C$ are consecutive terms of an arithmetic progression.

(3)

In an acute-angled triangle PQR the sides QR, PR and PQ have lengths p, q and r respectively.

(*c*) Prove that

$$\frac{p}{\sin P} = \frac{q}{\sin Q} = \frac{r}{\sin R}.$$

(3)

Given now that triangle PQR is such that p^2, q^2 and r^2 are three consecutive terms of an arithmetic progression,

(*d*) use the cosine rule to prove that $\dfrac{2\cos Q}{q} = \dfrac{\cos P}{p} + \dfrac{\cos R}{r}$.

(6)

(*e*) Using the results given in parts (*c*) and (*d*), prove that $\cot P$, $\cot Q$ and $\cot R$ are consecutive terms in an arithmetic progression.

(3)

2007, Question 6:

6.

Figure 2

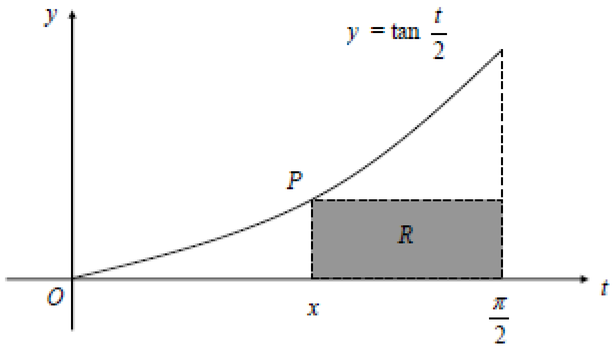

Figure 2 shows a sketch of the curve C with equation $y = \tan \frac{t}{2}$, $0 \leq t \leq \frac{\pi}{2}$.

The point P on C has coordinates $\left(x, \tan \frac{x}{2}\right)$.

The vertices of rectangle R are at $(x, 0)$, $\left(\frac{\pi}{2}, 0\right)$, $\left(\frac{\pi}{2}, \tan \frac{x}{2}\right)$ and $\left(x, \tan \frac{x}{2}\right)$ as shown in Figure 2.

(*a*) Find an expression, in terms of x, for the area A of R.

(1)

(*b*) Show that $\frac{dA}{dx} = \frac{1}{4}(\pi - 2x - 2 \sin x) \sec^2 \frac{x}{2}$.

(4)

(*c*) Prove that the maximum value of A occurs when $\frac{\pi}{4} < x < \frac{\pi}{3}$.

(7)

(*d*) Prove that $\tan \frac{\pi}{8} = \sqrt{2} - 1$.

(3)

(*e*) Show that the maximum value of $A > \frac{\pi}{4}(\sqrt{2} - 1)$.

(2)

2009, Question 2:

2. The curve C has equation $y = x^{\sin x}$, $x > 0$.

(a) Find the equation of the tangent to C at the point where $x = \frac{\pi}{2}$.

(6)

(b) Prove that this tangent touches C at infinitely many points.

(3)

2011, Question 1:

1. Solve for $0 \leqslant \theta \leqslant 180°$

$$\tan\left(\theta + 35°\right) = \cot\left(\theta - 53°\right)$$

(Total 4 marks)

2012, Question 7:

7. [arccos$x$ and arctan$x$ are alternative notation for $\cos^{-1}x$ and $\tan^{-1}x$ respectively]

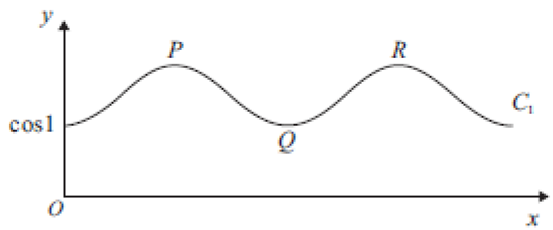

Figure 2

Figure 2 shows a sketch of the curve C_1 with equation $y = \cos(\cos x)$, $0 \leqslant x < 2\pi$.

The curve has turning points at (0, cos1), P, Q and R as shown in Figure 2.

(a) Find the coordinates of the points P, Q and R.

(4)

The curve C_2 has equation $y = \sin(\cos x)$, $0 \leqslant x < 2\pi$. The curves C_1 and C_2 intersect at the points S and T.

(b) Copy Figure 2 and on this diagram sketch C_2 stating the coordinates of the minimum point on C_2 and the points where C_2 meets or crosses the coordinate axes.

(5)

The coordinates of S are (α, d) where $0 < \alpha < \pi$.

(c) Show that $\alpha = \arccos\left(\frac{\pi}{4}\right)$.

(2)

(d) Find the value of d in surd form and write down the coordinates of T.

(3)

The tangent to C_1 at the point S has gradient $\tan\beta$.

(e) Show that $\beta = \arctan\sqrt{\left(\frac{16 - \pi^2}{32}\right)}$.

(5)

(f) Find, in terms of β, the obtuse angle between the tangent to C_1 at S and the tangent to C_2 at S.
(5)
