Core 3 Differentiation Questions
(From AEA Papers)

For answers, see [the AEA website]

2002, Question 5:

5. **Figure 1**

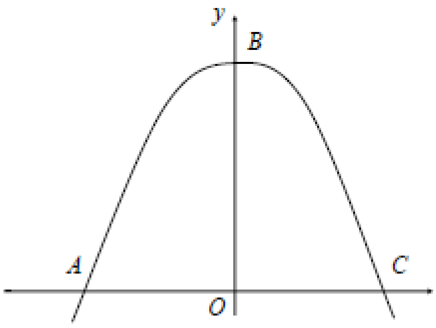

Figure 1 shows a sketch of part of the curve with equation

$$y = \sin(\cos x).$$

The curve cuts the x-axis at the points A and C and the y-axis at the point B.

(*a*) Find the coordinates of the points A, B and C. **(3)**

(*b*) Prove that B is a stationary point. **(2)**

Given that the region OCB is convex,

(*c*) show that, for $0 \leq x \leq \frac{\pi}{2}$,

$$\sin(\cos x) \leq \cos x$$

and

$$(1 - \frac{2}{\pi}x)\sin 1 \leq \sin(\cos x)$$

and state in each case the value or values of x for which equality is achieved. **(6)**

(*d*) Hence show that

$$\frac{\pi}{4}\sin 1 < \int_0^{\frac{\pi}{2}} \sin(\cos x) \, dx < 1.$$

(4)

2005, Question 3:

3. Given that

$$\frac{\mathrm{d}}{\mathrm{d}x}(u\sqrt{x}) = \frac{\mathrm{d}u}{\mathrm{d}x} \times \frac{\mathrm{d}(\sqrt{x})}{\mathrm{d}x}, \quad 0 < x < \tfrac{1}{2},$$

where u is a function of x, and that $u = 4$ when $x = \frac{3}{8}$, find u in terms of x.

(9)

2006, Question 6:

6.

Figure 1

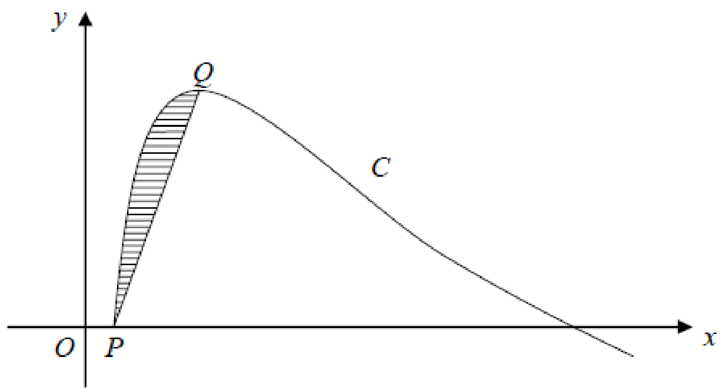

Figure 1 shows a sketch of part of the curve C with equation

$$y = \sin(\ln x), \quad x \geq 1.$$

The point Q, on C, is a maximum.

(a) Show that the point $P(1, 0)$ lies on C.

(1)

(b) Find the coordinates of the point Q.

(5)

(c) Find the area of the shaded region between C and the line PQ.

(9)

2009, Question 4:

4. (a) The function $f(x)$ has $f'(x) = \frac{u(x)}{v(x)}$. Given that $f'(k) = 0$,

show that $f''(k) = \frac{u'(k)}{v(k)}$.

(3)

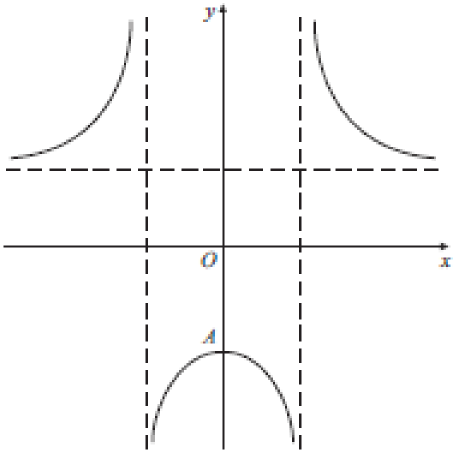

Figure 1

(b) The curve C with equation

$$y = \frac{2x^2+3}{x^2-1}$$

crosses the y-axis at the point A. Figure 1 shows a sketch of C together with its 3 asymptotes.

(i) Find the coordinates of the point A.

(1)

(ii) Find the equations of the asymptotes of C.

(2)

The point $P(a, b)$, $a > 0$ and $b > 0$, lies on C. The point Q also lies on C with PQ parallel to the x-axis and $AP = AQ$.

(iii) Show that the area of triangle PAQ is given by $\frac{5a^3}{a^2-1}$.

(2)

(iv) Find, as a varies, the minimum area of triangle PAQ, giving your answer in its simplest form.

(6)