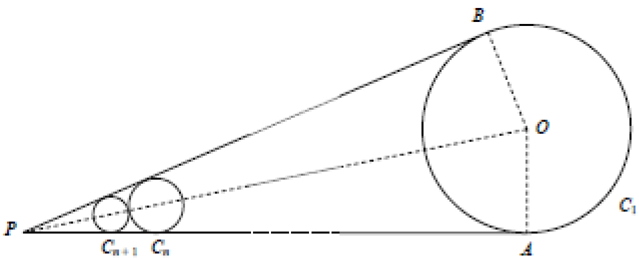
Core 2 Sequences Questions
(From AEA Papers)

For answers, see the AEA website

2006, Question 7:

7. Figure 2



The circle C_1 has centre O and radius R. The tangents AP and BP to C_1 meet at the point P and angle $APB = 2\alpha$, $0 < \alpha < \frac{\pi}{2}$. A sequence of circles $C_1, C_2, \ldots, C_n, \ldots$ is drawn so that each new circle C_{n+1} touches each of C_n, AP and BP for $n = 1, 2, 3, \ldots$ as shown in Figure 2. The centre of each circle lies on the line OP.

(*a*) Show that the radii of the circles form a geometric sequence with common ratio

$$\frac{1-\sin\alpha}{1+\sin\alpha}.$$

(5)

(*b*) Find, in terms of R and α, the total area enclosed by all the circles, simplifying your answer.

(3)

The area inside the quadrilateral $PAOB$, not enclosed by part of C_1 or any of the other circles, is S.

(*c*) Show that

$$S = R^2\left(\alpha + \cot\alpha - \frac{\pi}{4}\operatorname{cosec}\alpha - \frac{\pi}{4}\sin\alpha\right).$$

(5)

(*d*) Show that, as α varies,

$$\frac{dS}{d\alpha} = R^2\cot^2\alpha\left(\frac{\pi}{4}\cos\alpha - 1\right).$$

(4)

(*e*) Find, in terms of R, the least value of S for $\frac{\pi}{6} \le \alpha \le \frac{\pi}{4}$.

(3)

2007, Question 5:

5.

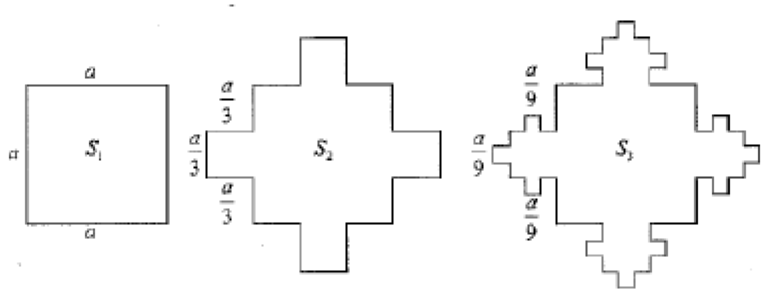


Figure 1

Figure 1 shows part of a sequence $S_1, S_2, S_3, \ldots$, of model snowflakes. The first term S_1 consists of a single square of side a. To obtain S_2, the middle third of each edge is replaced with a new square, of side $\frac{a}{3}$, as shown in Figure 1. Subsequent terms are obtained by replacing the middle third of each external edge of a new square formed in the previous snowflake, by a square $\frac{1}{3}$ of the size, as illustrated by S_3 in Figure 1.

(*a*) Deduce that to form S_4, 36 new squares of side $\frac{a}{27}$ must be added to S_3. **(1)**

(*b*) Show that the perimeters of S_2 and S_3 are $\frac{20a}{3}$ and $\frac{28a}{3}$ respectively. **(2)**

(*c*) Find the perimeter of S_n. **(4)**

(*d*) Describe what happens to the perimeter of S_n as n increases. **(1)**

(*e*) Find the areas of S_1, S_2 and S_3. **(2)**

(*f*) Find the smallest value of the constant S such that the area of $S_n < S$, for all values of n. **(5)**

2008, Question 1:

1. The first and second terms of an arithmetic series are 200 and 197.5 respectively.

The sum to n terms of the series is S_n.

Find the largest positive value of S_n.

(Total 5 marks)

2010, Question 2:

2. The sum of the first p terms of an arithmetic series is q and the sum of the first q terms of the same arithmetic series is p, where p and q are positive integers and $p \neq q$.

Giving simplified answers in terms of p and q, find

(a) the common difference of the terms in this series,

(5)

(b) the first term of the series,

(3)

(c) the sum of the first $(p+q)$ terms of the series.

(3)

2013, Question 4:

4. A sequence of positive integers $a_1, a_2, a_3, \ldots$ has rth term given by

$$a_r = 2^r - 1$$

(a) Write down the first 6 terms of this sequence.

(1)

(b) Verify that $a_{r+1} = 2a_r + 1$

(1)

(c) Find $\sum_{r=1}^{n} a_r$

(3)

(d) Show that $\frac{1}{a_{r+1}} < \frac{1}{2} \times \frac{1}{a_r}$

(1)

(e) Hence show that $1 + \frac{1}{3} + \frac{1}{7} + \frac{1}{15} + \frac{1}{31} + \ldots < 1 + \frac{1}{3} + \left(\frac{1}{7} + \frac{\frac{1}{2}}{7} + \frac{\frac{1}{4}}{7} + \ldots \right)$

(2)

(f) Show that $\frac{31}{21} < \sum_{r=1}^{\infty} \frac{1}{a_r} < \frac{34}{21}$

(5)
