

5. [In this question the values of a , x , and n are such that a and x are positive real numbers, with $a > 1$, $x \neq a$, $x \neq 1$ and n is an integer with $n > 1$]

Sam was confused about the rules of logarithms and thought that

$$\log_a x^n = (\log_a x)^n \quad (1)$$

- (a) Given that x satisfies statement (1) find x in terms of a and n .

(3)

Sam also thought that

$$\log_a x + \log_a x^2 + \dots + \log_a x^n = \log_a x + (\log_a x)^2 + \dots + (\log_a x)^n \quad (2)$$

- (b) For $n = 3$, x_1 and x_2 ($x_1 > x_2$) are the two values of x that satisfy statement (2).

- (i) Find, in terms of a , an expression for x_1 and an expression for x_2 .

(ii) Find the exact value of $\log_a \left(\frac{x_1}{x_2} \right)$.

(5)

- (c) Show that if $\log_a x$ satisfies statement (2) then

$$2(\log_a x)^n - n(n+1)\log_a x + (n^2 + n - 2) = 0$$

(6)