

5.

Figure 1

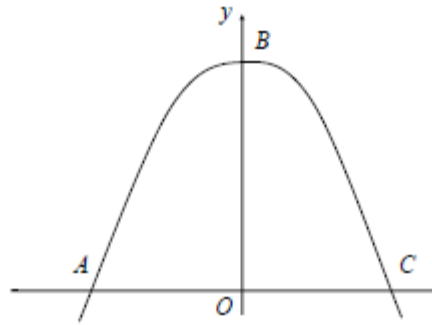


Figure 1 shows a sketch of part of the curve with equation

$$y = \sin(\cos x).$$

The curve cuts the x -axis at the points A and C and the y -axis at the point B .

(a) Find the coordinates of the points A , B and C .

(3)

(b) Prove that B is a stationary point.

(2)

Given that the region OCB is convex,

(c) show that, for $0 \leq x \leq \frac{\pi}{2}$,

$$\sin(\cos x) \leq \cos x$$

and

$$\left(1 - \frac{2}{\pi}x\right) \sin 1 \leq \sin(\cos x)$$

and state in each case the value or values of x for which equality is achieved.

(6)

(d) Hence show that

$$\frac{\pi}{4} \sin 1 < \int_0^{\frac{\pi}{2}} \sin(\cos x) \, dx < 1.$$

(4)