

OXFORD CAMBRIDGE AND RSA EXAMINATIONS

Advanced Subsidiary General Certificate of Education Advanced General Certificate of Education

MATHEMATICS 4725

Further Pure Mathematics 1

Thursday **8 JUNE 2006** Morning 1 hour 30 minutes

Additional materials: 8 page answer booklet Graph paper List of Formulae (MF1)

TIME 1 hour 30 minutes

INSTRUCTIONS TO CANDIDATES

- Write your name, centre number and candidate number in the spaces provided on the answer booklet.
- Answer all the questions.
- Give non-exact numerical answers correct to 3 significant figures unless a different degree of accuracy is specified in the question or is clearly appropriate.
- You are permitted to use a graphical calculator in this paper.

INFORMATION FOR CANDIDATES

- The number of marks is given in brackets [] at the end of each question or part question.
- The total number of marks for this paper is 72.
- Questions carrying smaller numbers of marks are printed earlier in the paper, and questions carrying larger numbers of marks later in the paper.
- You are reminded of the need for clear presentation in your answers.

1 The matrices **A** and **B** are given by $\mathbf{A} = \begin{pmatrix} 4 & 1 \\ 0 & 2 \end{pmatrix}$ and $\mathbf{B} = \begin{pmatrix} 1 & 1 \\ 0 & -1 \end{pmatrix}$.

(i) Find
$$A + 3B$$
. [2]

- (ii) Show that $\mathbf{A} \mathbf{B} = k\mathbf{I}$, where \mathbf{I} is the identity matrix and k is a constant whose value should be stated.
- 2 The transformation S is a shear parallel to the x-axis in which the image of the point (1, 1) is the point (0, 1).
 - (i) Draw a diagram showing the image of the unit square under S. [2]
 - (ii) Write down the matrix that represents S. [2]
- 3 One root of the quadratic equation $x^2 + px + q = 0$, where p and q are real, is the complex number 2-3i.
 - (i) Write down the other root. [1]
 - (ii) Find the values of p and q. [4]
- 4 Use the standard results for $\sum_{r=1}^{n} r^3$ and $\sum_{r=1}^{n} r^2$ to show that, for all positive integers n,

$$\sum_{r=1}^{n} (r^3 + r^2) = \frac{1}{12} n(n+1)(n+2)(3n+1).$$
 [5]

5 The complex numbers 3 - 2i and 2 + i are denoted by z and w respectively. Find, giving your answers in the form x + iy and showing clearly how you obtain these answers,

(i)
$$2z - 3w$$
, [2]

(ii)
$$(iz)^2$$
, [3]

(iii)
$$\frac{z}{w}$$
. [3]

6 In an Argand diagram the loci C_1 and C_2 are given by

$$|z| = 2$$
 and $\arg z = \frac{1}{3}\pi$

respectively.

- (i) Sketch, on a single Argand diagram, the loci C_1 and C_2 . [5]
- (ii) Hence find, in the form x + iy, the complex number representing the point of intersection of C_1 and C_2 . [2]

7 The matrix **A** is given by $\mathbf{A} = \begin{pmatrix} 2 & 0 \\ 0 & 1 \end{pmatrix}$.

(i) Find
$$A^2$$
 and A^3 .

- (ii) Hence suggest a suitable form for the matrix A^n . [1]
- (iii) Use induction to prove that your answer to part (ii) is correct. [4]
- 8 The matrix **M** is given by $\mathbf{M} = \begin{pmatrix} a & 4 & 2 \\ 1 & a & 0 \\ 1 & 2 & 1 \end{pmatrix}$.
 - (i) Find, in terms of a, the determinant of M. [3]
 - (ii) Hence find the values of a for which M is singular. [3]
 - (iii) State, giving a brief reason in each case, whether the simultaneous equations

$$ax + 4y + 2z = 3a,$$

$$x + ay = 1,$$

$$x + 2y + z = 3,$$

have any solutions when

- (a) a = 3,
- **(b)** a = 2.

[4]

9 (i) Use the method of differences to show that

$$\sum_{r=1}^{n} \left\{ (r+1)^3 - r^3 \right\} = (n+1)^3 - 1.$$
 [2]

- (ii) Show that $(r+1)^3 r^3 \equiv 3r^2 + 3r + 1$. [2]
- (iii) Use the results in parts (i) and (ii) and the standard result for $\sum_{r=1}^{n} r$ to show that

$$3\sum_{r=1}^{n}r^{2} = \frac{1}{2}n(n+1)(2n+1).$$
 [6]

10 The cubic equation $x^3 - 2x^2 + 3x + 4 = 0$ has roots α , β and γ .

(i) Write down the values of
$$\alpha + \beta + \gamma$$
, $\alpha\beta + \beta\gamma + \gamma\alpha$ and $\alpha\beta\gamma$. [3]

The cubic equation $x^3 + px^2 + 10x + q = 0$, where p and q are constants, has roots $\alpha + 1$, $\beta + 1$ and $\gamma + 1$.

(ii) Find the value of
$$p$$
. [3]

(iii) Find the value of
$$q$$
. [5]

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