

**OXFORD CAMBRIDGE AND RSA EXAMINATIONS**

**Advanced Subsidiary General Certificate of Education  
Advanced General Certificate of Education**

**MATHEMATICS**

**4723**

Core Mathematics 3

Thursday

**8 JUNE 2006**

Morning

1 hour 30 minutes

Additional materials:

8 page answer booklet

Graph paper

List of Formulae (MF1)

**TIME** 1 hour 30 minutes

**INSTRUCTIONS TO CANDIDATES**

- Write your name, centre number and candidate number in the spaces provided on the answer booklet.
- Answer **all** the questions.
- Give non-exact numerical answers correct to 3 significant figures unless a different degree of accuracy is specified in the question or is clearly appropriate.
- You are permitted to use a graphical calculator in this paper.

**INFORMATION FOR CANDIDATES**

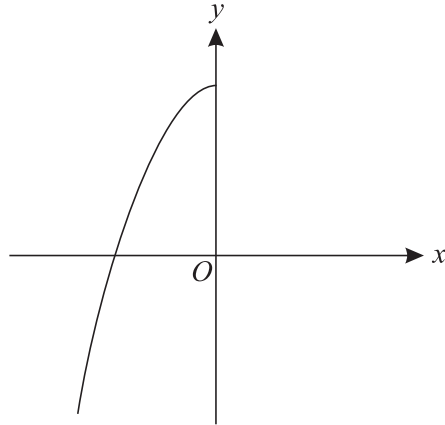
- The number of marks is given in brackets [ ] at the end of each question or part question.
- The total number of marks for this paper is 72.
- Questions carrying smaller numbers of marks are printed earlier in the paper, and questions carrying larger numbers of marks later in the paper.
- **You are reminded of the need for clear presentation in your answers.**

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**This question paper consists of 4 printed pages.**

- 1 Find the equation of the tangent to the curve  $y = \sqrt{4x + 1}$  at the point (2, 3). [5]
- 2 Solve the inequality  $|2x - 3| < |x + 1|$ . [5]
- 3 The equation  $2x^3 + 4x - 35 = 0$  has one real root.
- (i) Show by calculation that this real root lies between 2 and 3. [3]
- (ii) Use the iterative formula
- $$x_{n+1} = \sqrt[3]{17.5 - 2x_n},$$
- with a suitable starting value, to find the real root of the equation  $2x^3 + 4x - 35 = 0$  correct to 2 decimal places. You should show the result of each iteration. [3]
- 4 It is given that  $y = 5^{x-1}$ .
- (i) Show that  $x = 1 + \frac{\ln y}{\ln 5}$ . [2]
- (ii) Find an expression for  $\frac{dx}{dy}$  in terms of  $y$ . [2]
- (iii) Hence find the exact value of the gradient of the curve  $y = 5^{x-1}$  at the point (3, 25). [2]
- 5 (i) Write down the identity expressing  $\sin 2\theta$  in terms of  $\sin \theta$  and  $\cos \theta$ . [1]
- (ii) Given that  $\sin \alpha = \frac{1}{4}$  and  $\alpha$  is acute, show that  $\sin 2\alpha = \frac{1}{8}\sqrt{15}$ . [3]
- (iii) Solve, for  $0^\circ < \beta < 90^\circ$ , the equation  $5 \sin 2\beta \sec \beta = 3$ . [3]

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The diagram shows the graph of  $y = f(x)$ , where

$$f(x) = 2 - x^2, \quad x \leq 0.$$

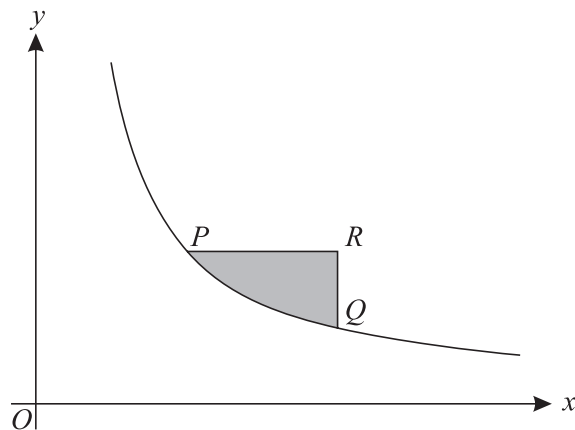
(i) Evaluate  $ff(-3)$ . [3]

(ii) Find an expression for  $f^{-1}(x)$ . [3]

(iii) Sketch the graph of  $y = f^{-1}(x)$ . Indicate the coordinates of the points where the graph meets the axes. [3]

7 (a) Find the exact value of  $\int_1^2 \frac{2}{(4x-1)^2} dx$ . [4]

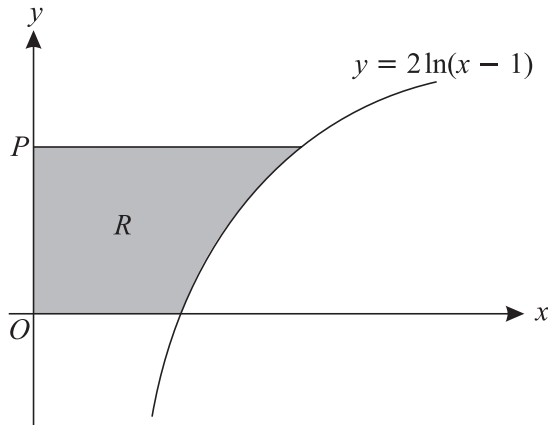
(b)



The diagram shows part of the curve  $y = \frac{1}{x}$ . The point  $P$  has coordinates  $(a, \frac{1}{a})$  and the point  $Q$  has coordinates  $(2a, \frac{1}{2a})$ , where  $a$  is a positive constant. The point  $R$  is such that  $PR$  is parallel to the  $x$ -axis and  $QR$  is parallel to the  $y$ -axis. The region shaded in the diagram is bounded by the curve and by the lines  $PR$  and  $QR$ . Show that the area of this shaded region is  $\ln(\frac{1}{2}e)$ . [6]

- 8 (i) Express  $5 \cos x + 12 \sin x$  in the form  $R \cos(x - \alpha)$ , where  $R > 0$  and  $0^\circ < \alpha < 90^\circ$ . [3]
- (ii) Hence give details of a pair of transformations which transforms the curve  $y = \cos x$  to the curve  $y = 5 \cos x + 12 \sin x$ . [3]
- (iii) Solve, for  $0^\circ < x < 360^\circ$ , the equation  $5 \cos x + 12 \sin x = 2$ , giving your answers correct to the nearest  $0.1^\circ$ . [5]

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The diagram shows the curve with equation  $y = 2 \ln(x - 1)$ . The point  $P$  has coordinates  $(0, p)$ . The region  $R$ , shaded in the diagram, is bounded by the curve and the lines  $x = 0$ ,  $y = 0$  and  $y = p$ . The units on the axes are centimetres. The region  $R$  is rotated completely about the **y-axis** to form a solid.

- (i) Show that the volume,  $V \text{ cm}^3$ , of the solid is given by

$$V = \pi(e^p + 4e^{\frac{1}{2}p} + p - 5). \quad [8]$$

- (ii) It is given that the point  $P$  is moving in the positive direction along the  $y$ -axis at a constant rate of  $0.2 \text{ cm min}^{-1}$ . Find the rate at which the volume of the solid is increasing at the instant when  $p = 4$ , giving your answer correct to 2 significant figures. [5]