

**OXFORD CAMBRIDGE AND RSA EXAMINATIONS**

**Advanced Subsidiary General Certificate of Education  
Advanced General Certificate of Education**

**MATHEMATICS**

**4725**

Further Pure Mathematics 1

Tuesday

**7 JUNE 2005**

Afternoon

1 hour 30 minutes

Additional materials:

- Answer booklet
- Graph paper
- List of Formulae (MF1)

**TIME** 1 hour 30 minutes

**INSTRUCTIONS TO CANDIDATES**

- Write your name, centre number and candidate number in the spaces provided on the answer booklet.
- Answer **all** the questions.
- Give non-exact numerical answers correct to 3 significant figures unless a different degree of accuracy is specified in the question or is clearly appropriate.
- You are permitted to use a graphical calculator in this paper.

**INFORMATION FOR CANDIDATES**

- The number of marks is given in brackets [ ] at the end of each question or part question.
- The total number of marks for this paper is 72.
- Questions carrying smaller numbers of marks are printed earlier in the paper, and questions carrying larger numbers of marks later in the paper.
- **You are reminded of the need for clear presentation in your answers.**

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**This question paper consists of 3 printed pages and 1 blank page.**

- 1 Use the standard results for  $\sum_{r=1}^n r$  and  $\sum_{r=1}^n r^2$  to show that, for all positive integers  $n$ ,

$$\sum_{r=1}^n (6r^2 + 2r + 1) = n(2n^2 + 4n + 3). \quad [6]$$

- 2 The matrices  $\mathbf{A}$  and  $\mathbf{I}$  are given by  $\mathbf{A} = \begin{pmatrix} 1 & 2 \\ 1 & 3 \end{pmatrix}$  and  $\mathbf{I} = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$  respectively.

(i) Find  $\mathbf{A}^2$  and verify that  $\mathbf{A}^2 = 4\mathbf{A} - \mathbf{I}$ . [4]

(ii) Hence, or otherwise, show that  $\mathbf{A}^{-1} = 4\mathbf{I} - \mathbf{A}$ . [2]

- 3 The complex numbers  $2 + 3i$  and  $4 - i$  are denoted by  $z$  and  $w$  respectively. Express each of the following in the form  $x + iy$ , showing clearly how you obtain your answers.

(i)  $z + 5w$ , [2]

(ii)  $z^*w$ , where  $z^*$  is the complex conjugate of  $z$ , [3]

(iii)  $\frac{1}{w}$ . [2]

- 4 Use an algebraic method to find the square roots of the complex number  $21 - 20i$ . [6]

- 5 (i) Show that

$$\frac{r+1}{r+2} - \frac{r}{r+1} = \frac{1}{(r+1)(r+2)}. \quad [2]$$

- (ii) Hence find an expression, in terms of  $n$ , for

$$\frac{1}{6} + \frac{1}{12} + \frac{1}{20} + \dots + \frac{1}{(n+1)(n+2)}. \quad [4]$$

(iii) Hence write down the value of  $\sum_{r=1}^{\infty} \frac{1}{(r+1)(r+2)}$ . [1]

- 6 The loci  $C_1$  and  $C_2$  are given by

$$|z - 2i| = 2 \quad \text{and} \quad |z + 1| = |z + i|$$

respectively.

(i) Sketch, on a single Argand diagram, the loci  $C_1$  and  $C_2$ . [5]

(ii) Hence write down the complex numbers represented by the points of intersection of  $C_1$  and  $C_2$ . [2]

7 The matrix  $\mathbf{B}$  is given by  $\mathbf{B} = \begin{pmatrix} a & 1 & 3 \\ 2 & 1 & -1 \\ 0 & 1 & 2 \end{pmatrix}$ .

(i) Given that  $\mathbf{B}$  is singular, show that  $a = -\frac{2}{3}$ . [3]

(ii) Given instead that  $\mathbf{B}$  is non-singular, find the inverse matrix  $\mathbf{B}^{-1}$ . [4]

(iii) Hence, or otherwise, solve the equations

$$\begin{aligned} -x + y + 3z &= 1, \\ 2x + y - z &= 4, \\ y + 2z &= -1. \end{aligned} \quad [3]$$

8 (a) The quadratic equation  $x^2 - 2x + 4 = 0$  has roots  $\alpha$  and  $\beta$ .

(i) Write down the values of  $\alpha + \beta$  and  $\alpha\beta$ . [2]

(ii) Show that  $\alpha^2 + \beta^2 = -4$ . [2]

(iii) Hence find a quadratic equation which has roots  $\alpha^2$  and  $\beta^2$ . [3]

(b) The cubic equation  $x^3 - 12x^2 + ax - 48 = 0$  has roots  $p$ ,  $2p$  and  $3p$ .

(i) Find the value of  $p$ . [2]

(ii) Hence find the value of  $a$ . [2]

9 (i) Write down the matrix  $\mathbf{C}$  which represents a stretch, scale factor 2, in the  $x$ -direction. [2]

(ii) The matrix  $\mathbf{D}$  is given by  $\mathbf{D} = \begin{pmatrix} 1 & 3 \\ 0 & 1 \end{pmatrix}$ . Describe fully the geometrical transformation represented by  $\mathbf{D}$ . [2]

(iii) The matrix  $\mathbf{M}$  represents the combined effect of the transformation represented by  $\mathbf{C}$  followed by the transformation represented by  $\mathbf{D}$ . Show that

$$\mathbf{M} = \begin{pmatrix} 2 & 3 \\ 0 & 1 \end{pmatrix}. \quad [2]$$

(iv) Prove by induction that  $\mathbf{M}^n = \begin{pmatrix} 2^n & 3(2^n - 1) \\ 0 & 1 \end{pmatrix}$ , for all positive integers  $n$ . [6]

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