

Mechanics 2 Differential Equations

- 6 A student is modelling the motion of a small boat as it moves on a lake. When the speed of the boat is 12 m s^{-1} , the engine is switched off. At time t seconds later, it has a velocity of $v \text{ m s}^{-1}$ and experiences a resistance force of magnitude $20v$ newtons. The mass of the boat is 80 kg .

To set up a simple model for the motion of the boat, the student assumes that the water in the lake is still and that the boat travels in a straight line.

- (a) Explain how these two assumptions allow the student to create a simple model. *(2 marks)*
- (b) State one other assumption that the student should make. *(1 mark)*
- (c) (i) Express $\frac{dv}{dt}$ in terms of v . *(2 marks)*
- (ii) Find an expression for v in terms of t . *(5 marks)*
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- 7 A particle of mass 20 kg moves along a straight horizontal line. At time t seconds the velocity of the particle is $v \text{ m s}^{-1}$. A resistance force of magnitude $10\sqrt{v}$ newtons acts on the particle while it is moving. At time $t = 0$ the velocity of the particle is 25 m s^{-1} .

- (a) Show that, at time t

$$v = \left(\frac{20 - t}{4} \right)^2 \quad (7 \text{ marks})$$

- (b) State the value of t when the particle comes to rest. *(1 mark)*
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7 A motorcycle has a maximum power of 72 kilowatts. The motorcycle and its rider are travelling along a straight horizontal road. When they are moving at a speed of $V \text{ m s}^{-1}$, they experience a total resistance force of magnitude kV newtons, where k is a constant.

(a) The maximum speed of the motorcycle and its rider is 60 m s^{-1} .

Show that $k = 20$. (3 marks)

(b) When the motorcycle is travelling at 20 m s^{-1} , the rider allows the motorcycle to freewheel so that the only horizontal force acting is the resistance force. When the motorcycle has been freewheeling for t seconds, its speed is $v \text{ m s}^{-1}$ and the magnitude of the resistance force is $20v$ newtons.

The mass of the motorcycle and its rider is 500 kg.

(i) Show that $\frac{dv}{dt} = -\frac{v}{25}$. (2 marks)

(ii) Hence find the time that it takes for the speed of the motorcycle to reduce from 20 m s^{-1} to 10 m s^{-1} . (6 marks)

7 A stone of mass m is moving along the smooth horizontal floor of a tank which is filled with a viscous liquid. At time t , the stone has speed v . As the stone moves, it experiences a resistance force of magnitude $\lambda m v$, where λ is a constant.

(a) Show that

$$\frac{dv}{dt} = -\lambda v \quad (2 \text{ marks})$$

(b) The initial speed of the stone is U .

Show that

$$v = Ue^{-\lambda t} \quad (4 \text{ marks})$$
