FP3 Numerical Methods for the Solution of First Order Differential Equations Questions

5 (a) The function $y(x)$ satisfies the differential equation

$$\frac{dy}{dx} = f(x, y)$$

where $$f(x, y) = x \ln x + \frac{y}{x}$$

and $$y(1) = 1$$

(i) Use the Euler formula

$$y_{r+1} = y_r + h\, f(x_r, y_r)$$

with $h = 0.1$, to obtain an approximation to $y(1.1)$. *(3 marks)*

(ii) Use the formula

$$y_{r+1} = y_{r-1} + 2h\, f(x_r, y_r)$$

with your answer to part (a)(i) to obtain an approximation to $y(1.2)$, giving your answer to three decimal places. *(4 marks)*

(b) (i) Show that $\frac{1}{x}$ is an integrating factor for the first-order differential equation

$$\frac{dy}{dx} - \frac{1}{x}y = x \ln x$$

(3 marks)

(ii) Solve this differential equation, given that $y = 1$ when $x = 1$. *(6 marks)*

(iii) Calculate the value of y when $x = 1.2$, giving your answer to three decimal places. *(1 mark)*

2 The function $y(x)$ satisfies the differential equation

$$\frac{dy}{dx} = f(x, y)$$

where $$f(x, y) = \frac{x^2 + y^2}{xy}$$

and $$y(1) = 2$$

(a) Use the Euler formula

$$y_{r+1} = y_r + hf(x_r, y_r)$$

with $h = 0.1$, to obtain an approximation to $y(1.1)$. *(3 marks)*

(b) Use the improved Euler formula

$$y_{r+1} = y_r + \tfrac{1}{2}(k_1 + k_2)$$

where $k_1 = hf(x_r, y_r)$ and $k_2 = hf(x_r + h, y_r + k_1)$ and $h = 0.1$, to obtain an approximation to $y(1.1)$, giving your answer to four decimal places. *(6 marks)*

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1 The function $y(x)$ satisfies the differential equation

$$\frac{dy}{dx} = f(x, y)$$

where $$f(x, y) = \ln(1 + x^2 + y)$$

and $$y(1) = 0.6$$

(a) Use the Euler formula

$$y_{r+1} = y_r + h\,f(x_r, y_r)$$

with $h = 0.05$, to obtain an approximation to $y(1.05)$, giving your answer to four decimal places. *(3 marks)*

(b) Use the improved Euler formula

$$y_{r+1} = y_r + \tfrac{1}{2}(k_1 + k_2)$$

where $k_1 = h\,f(x_r, y_r)$ and $k_2 = h\,f(x_r + h, y_r + k_1)$ and $h = 0.05$, to obtain an approximation to $y(1.05)$, giving your answer to four decimal places. *(6 marks)*

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2 The function $y(x)$ satisfies the differential equation

$$\frac{\mathrm{d}y}{\mathrm{d}x} = \mathrm{f}(x, y)$$

where $$\mathrm{f}(x, y) = \sqrt{x^2 + y^2 + 3}$$

and $$y(1) = 2$$

(a) Use the Euler formula

$$y_{r+1} = y_r + h\mathrm{f}(x_r, y_r)$$

with $h = 0.1$, to obtain an approximation to $y(1.1)$, giving your answer to four decimal places. *(3 marks)*

(b) Use the improved Euler formula

$$y_{r+1} = y_r + \tfrac{1}{2}(k_1 + k_2)$$

where $k_1 = h\mathrm{f}(x_r, y_r)$ and $k_2 = h\mathrm{f}(x_r + h, y_r + k_1)$ and $h = 0.1$, to obtain an approximation to $y(1.1)$, giving your answer to four decimal places. *(6 marks)*