

FP3 Introduction to Differential Equations Questions

- 3 (a) Show that $y = x^3 - x$ is a particular integral of the differential equation

$$\frac{dy}{dx} + \frac{2xy}{x^2 - 1} = 5x^2 - 1 \quad (3 \text{ marks})$$

- (b) By differentiating $(x^2 - 1)y = c$ implicitly, where y is a function of x and c is a constant, show that $y = \frac{c}{x^2 - 1}$ is a solution of the differential equation

$$\frac{dy}{dx} + \frac{2xy}{x^2 - 1} = 0 \quad (3 \text{ marks})$$

- (c) Hence find the general solution of

$$\frac{dy}{dx} + \frac{2xy}{x^2 - 1} = 5x^2 - 1 \quad (2 \text{ marks})$$

- 3 (a) Show that $\sin x$ is an integrating factor for the differential equation

$$\frac{dy}{dx} + (\cot x)y = 2 \cos x \quad (3 \text{ marks})$$

- (b) Solve this differential equation, given that $y = 2$ when $x = \frac{\pi}{2}$. (6 marks)
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- 3 (a) Show that x^2 is an integrating factor for the first-order differential equation

$$\frac{dy}{dx} + \frac{2}{x}y = 3(x^3 + 1)^{\frac{1}{2}} \quad (3 \text{ marks})$$

- (b) Solve this differential equation, given that $y = 1$ when $x = 2$. (6 marks)
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- 3 By using an integrating factor, find the solution of the differential equation

$$\frac{dy}{dx} + (\tan x)y = \sec x$$

- given that $y = 3$ when $x = 0$. (8 marks)
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