

FP1 Calculus Questions

- 2 (a) For each of the following improper integrals, find the value of the integral **or** explain briefly why it does not have a value:

(i) $\int_0^9 \frac{1}{\sqrt{x}} dx;$ (3 marks)

(ii) $\int_0^9 \frac{1}{x\sqrt{x}} dx.$ (3 marks)

- (b) Explain briefly why the integrals in part (a) are improper integrals. (1 mark)
-

- 8 (a) The function f is defined for all real values of x by

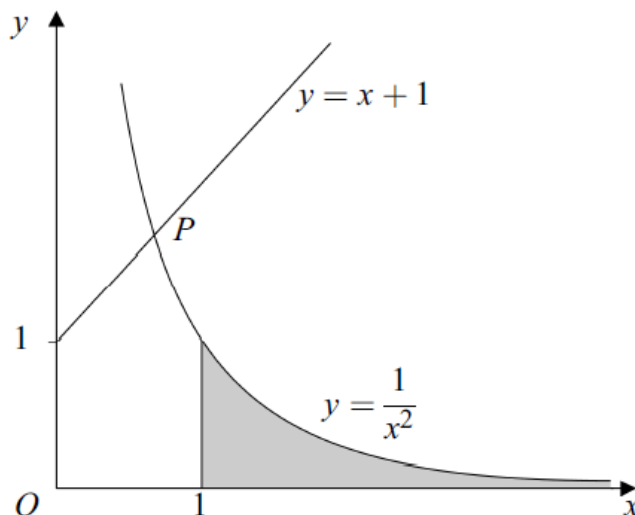
$$f(x) = x^3 + x^2 - 1$$

- (i) Express $f(1+h) - f(1)$ in the form

$$ph + qh^2 + rh^3$$

where p , q and r are integers. (4 marks)

- (ii) Use your answer to part (a)(i) to find the value of $f'(1)$. (2 marks)



- (c) The region enclosed by the curve $y = \frac{1}{x^2}$, the line $x = 1$ and the x -axis is shaded on the diagram. By evaluating an improper integral, find the area of this region. (3 marks)
-

The function f is defined for all real numbers by

$$f(x) = \sin\left(x + \frac{\pi}{6}\right)$$

(b) The quadratic function g is defined for all real numbers by

$$g(x) = \frac{1}{2} + \frac{\sqrt{3}}{2}x - \frac{1}{4}x^2$$

It can be shown that $g(x)$ gives a good approximation to $f(x)$ for small values of x .

- (i) Show that $g(0.05)$ and $f(0.05)$ are identical when rounded to four decimal places. *(2 marks)*
- (ii) A chord joins the points on the curve $y = g(x)$ for which $x = 0$ and $x = h$. Find an expression in terms of h for the gradient of this chord. *(2 marks)*
- (iii) Using your answer to part (b)(ii), find the value of $g'(0)$. *(1 mark)*

8 For each of the following improper integrals, find the value of the integral **or** explain briefly why it does not have a value:

(a) $\int_0^1 (x^{\frac{1}{3}} + x^{-\frac{1}{3}}) dx$; *(4 marks)*

(b) $\int_0^1 \frac{x^{\frac{1}{3}} + x^{-\frac{1}{3}}}{x} dx$. *(4 marks)*