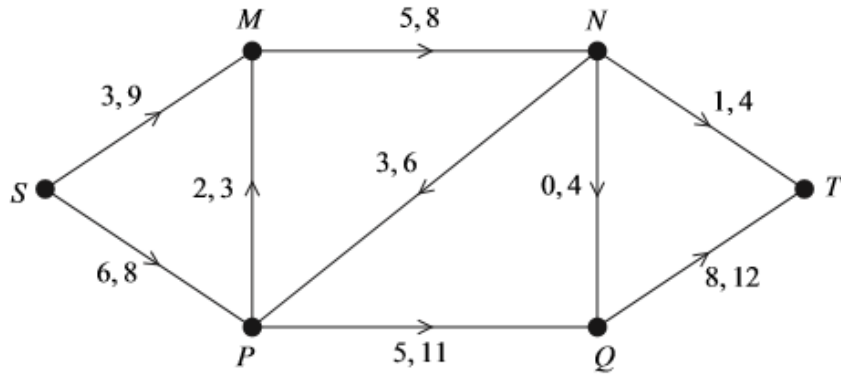


Decision 2 Network Flow Questions

4 [Figures 3, 4 and 5, printed on the insert, are provided for use in this question.]

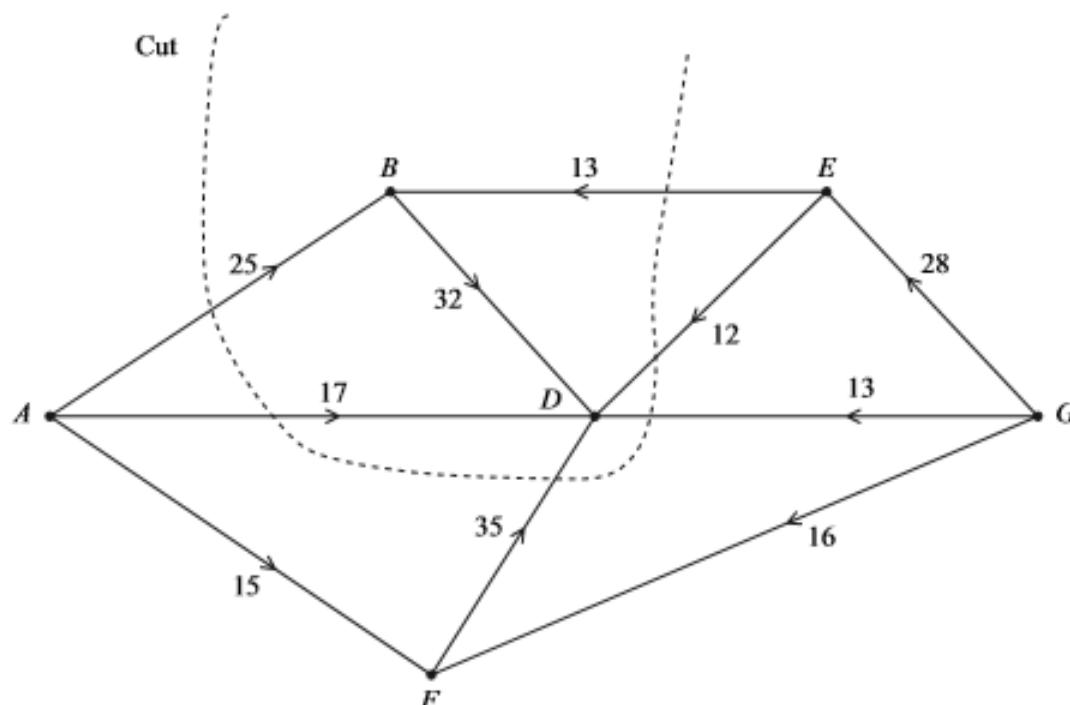
The network shows a system of pipes, with the lower and upper capacities for each pipe in litres per second.



- (a) **Figure 3**, on the insert, shows a partially completed diagram for a feasible flow of 10 litres per second from S to T . Indicate, on **Figure 3**, the flows along the edges MN , PQ , NP and NT . (4 marks)
- (b) (i) Taking your answer from part (a) as an initial flow, use flow augmentation on **Figure 4** to find the maximum flow from S to T . (6 marks)
- (ii) State the value of the maximum flow and illustrate this flow on **Figure 5**. (2 marks)
- (c) Find a cut with capacity equal to that of the maximum flow. (2 marks)

4 [Figures 4 and 5, printed on the insert, are provided for use in this question.]

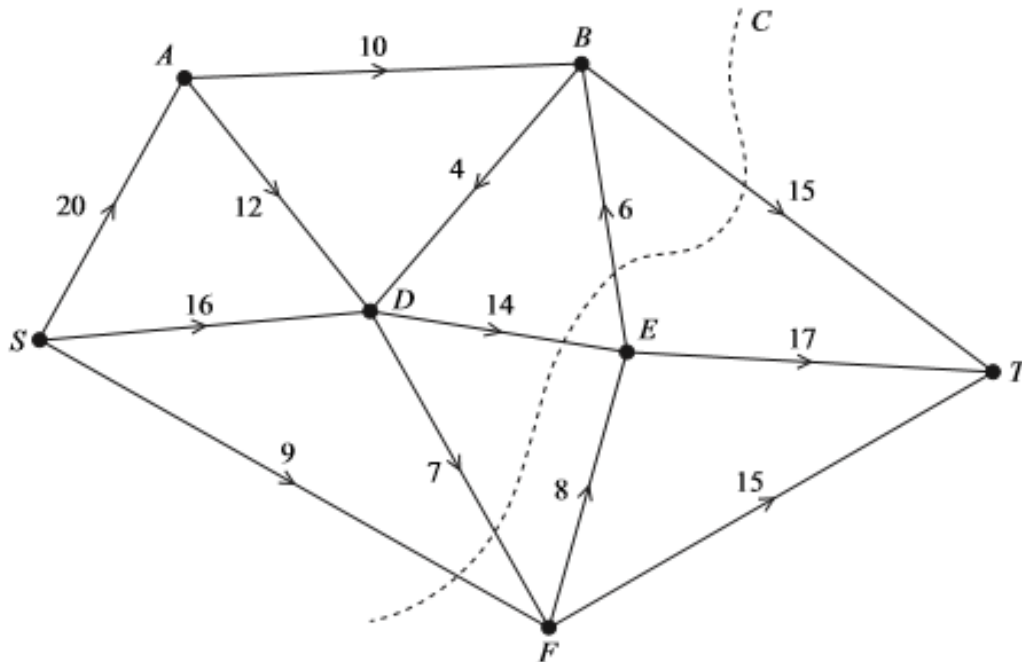
The network shows the routes along corridors from the playgrounds A and G to the assembly hall in a school. The number on each edge represents the maximum number of pupils that can travel along the corridor in one minute.



- (a) State the vertex that represents the assembly hall. (1 mark)
- (b) Find the value of the cut shown on the diagram. (1 mark)
- (c) State the maximum flow along the routes ABD and GED . (2 marks)
- (d) (i) Taking your answers to part (c) as the initial flow, use a labelling procedure on **Figure 4** to find the maximum flow through the network. (6 marks)
- (ii) State the value of the maximum flow and, on **Figure 5**, illustrate a possible flow along each edge corresponding to this maximum flow. (2 marks)
- (iii) Verify that your flow is a maximum flow by finding a cut of the same value. (2 marks)
- (e) On a particular day, there is an obstruction allowing no more than 15 pupils per minute to pass through vertex E . State the maximum number of pupils that can move through the network per minute on this particular day. (2 marks)

6 [Figures 2 and 3, printed on the insert, are provided for use in this question.]

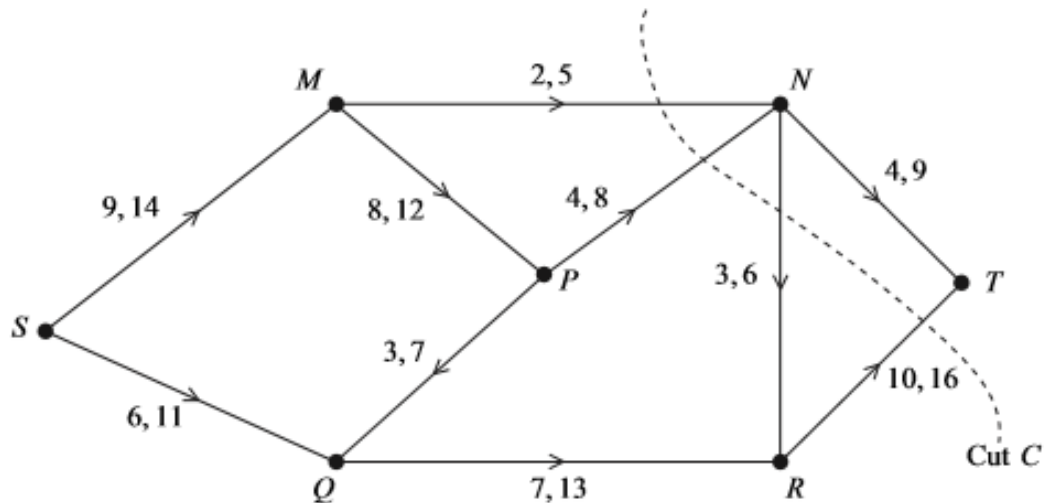
The diagram shows a network of pipelines through which oil can travel. The oil field is at S , the refinery is at T and the other vertices are intermediate stations. The weights on the edges show the capacities in millions of barrels per hour that can flow through each pipeline.



- (a) (i) Find the value of the cut marked C on the diagram. (1 mark)
- (ii) Hence make a deduction about the maximum flow of oil through the network. (2 marks)
- (b) State the maximum possible flows along the routes $SABT$, $SDET$ and SFT . (2 marks)
- (c) (i) Taking your answer to part (b) as the initial flow, use a labelling procedure on **Figure 2** to find the maximum flow from S to T . Record your routes and flows in the table provided and show the augmented flows on the network diagram. (6 marks)
- (ii) State the value of the maximum flow, and, on **Figure 3**, illustrate a possible flow along each edge corresponding to this maximum flow. (2 marks)
- (iii) Prove that your flow in part (c)(ii) is a maximum. (2 marks)
-

6 [Figures 4, 5 and 6, printed on the insert, are provided for use in this question.]

The network shows a system of pipes with the lower and upper capacities for each pipe in litres per second.



- (a) (i) Find the value of the cut C . (1 mark)
- (ii) State what can be deduced about the maximum flow from S to T . (1 mark)
- (b) **Figure 4**, printed on the insert, shows a partially completed diagram for a feasible flow of 20 litres per second from S to T . Indicate, on **Figure 4**, the flows along the edges MP , PN , QR and NR . (4 marks)
- (c) (i) Taking your answer from part (b) as an initial flow, indicate potential increases and decreases of the flow along each edge on **Figure 5**. (2 marks)
- (ii) Use flow augmentation on **Figure 5** to find the maximum flow from S to T . You should indicate any flow augmenting paths in the table and modify the potential increases and decreases of the flow on the network. (5 marks)
- (iii) Illustrate the maximum flow on **Figure 6**. (2 marks)
-

Figure 3 (for use in part (a))

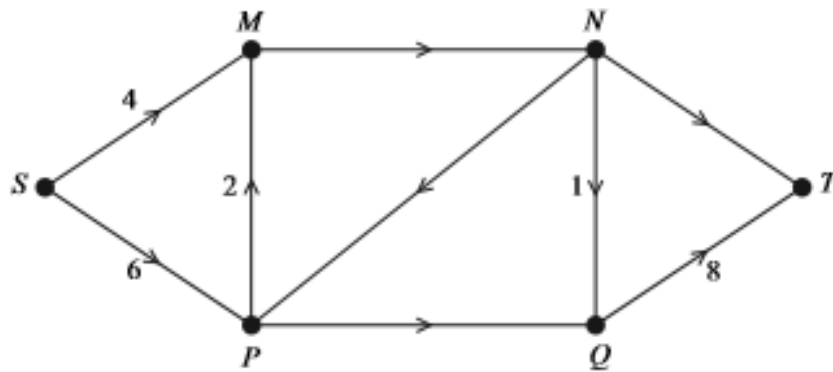
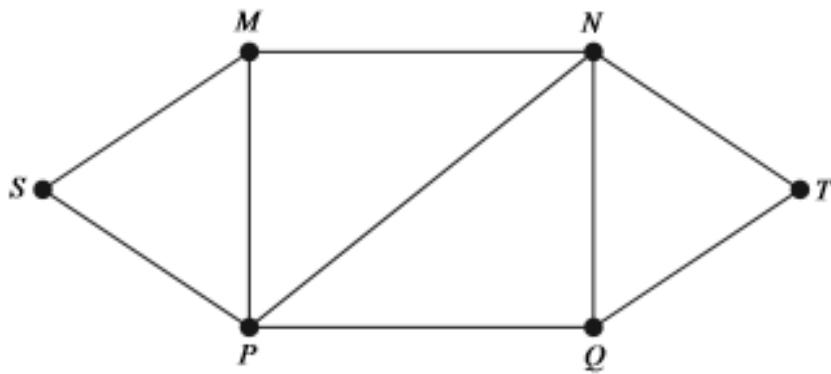


Figure 4 (for use in part (b)(i))



Route	Extra flow

Figure 5 (for use in part (b)(ii))

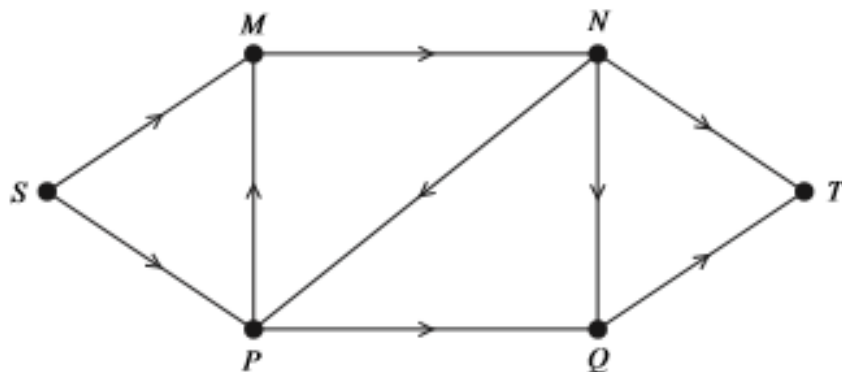
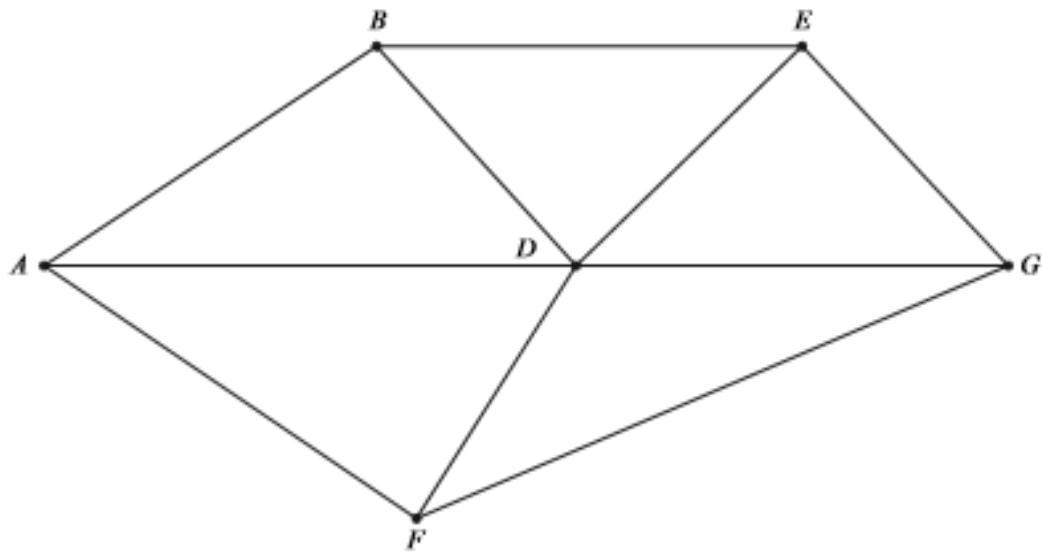


Figure 4 (for use in Question 4 part (d)(i))



Route	Flow
<i>ABD</i>	
<i>GED</i>	

Figure 5 (for use in Question 4 part (d)(ii))

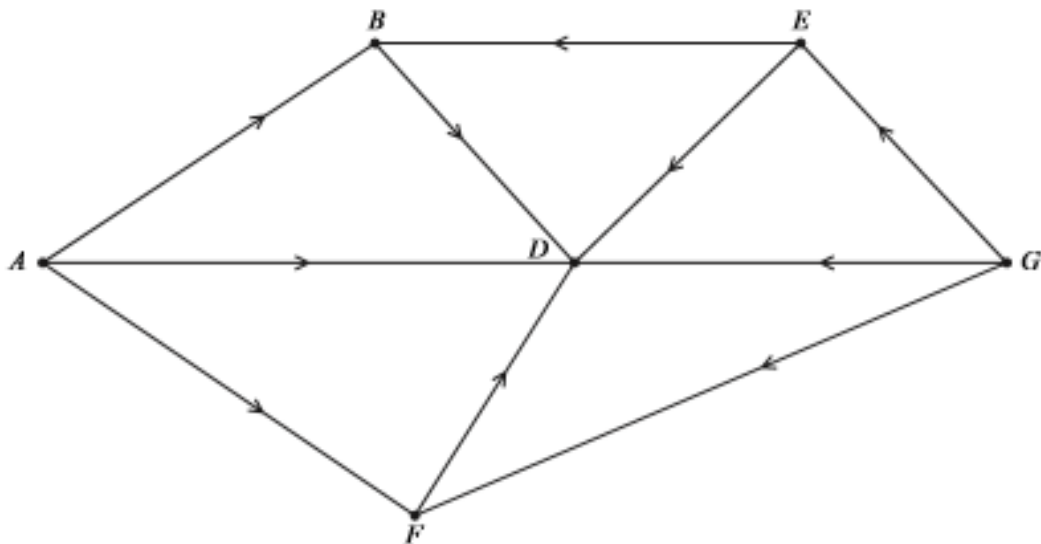
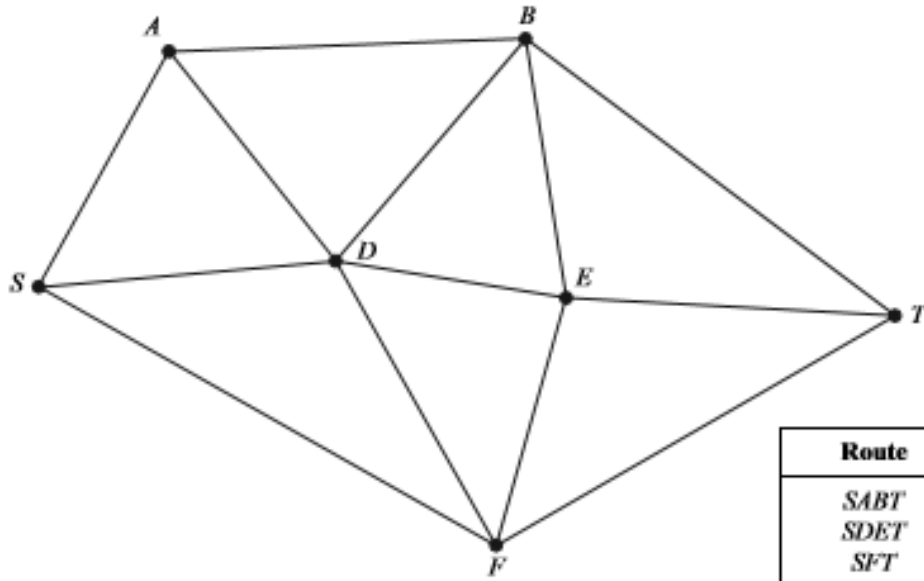


Figure 2 (for use in Question 6(c)(i))



Route	Flow
<i>SABT</i>	
<i>SDET</i>	
<i>SFT</i>	

Figure 3 (for use in Question 6(c)(ii))

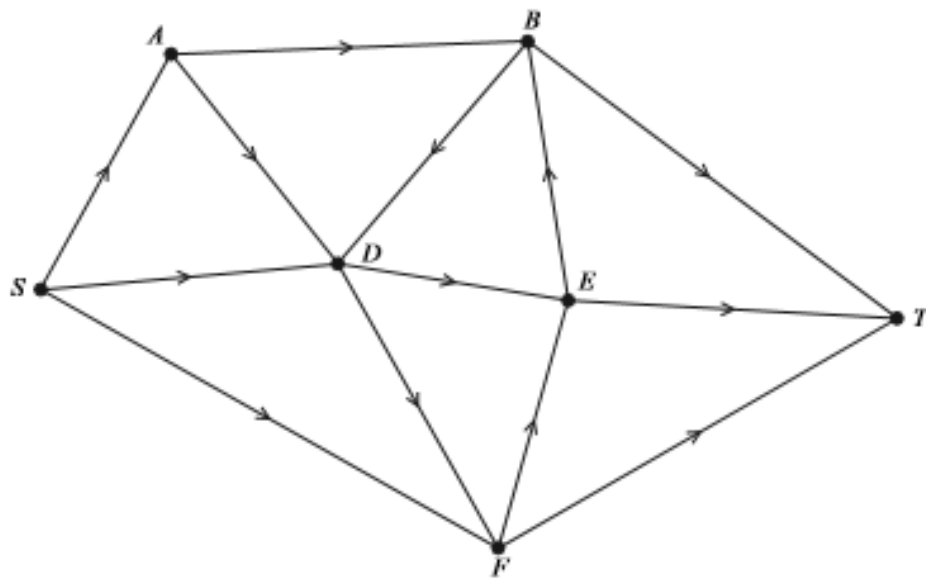


Figure 4 (for use in Question 6)

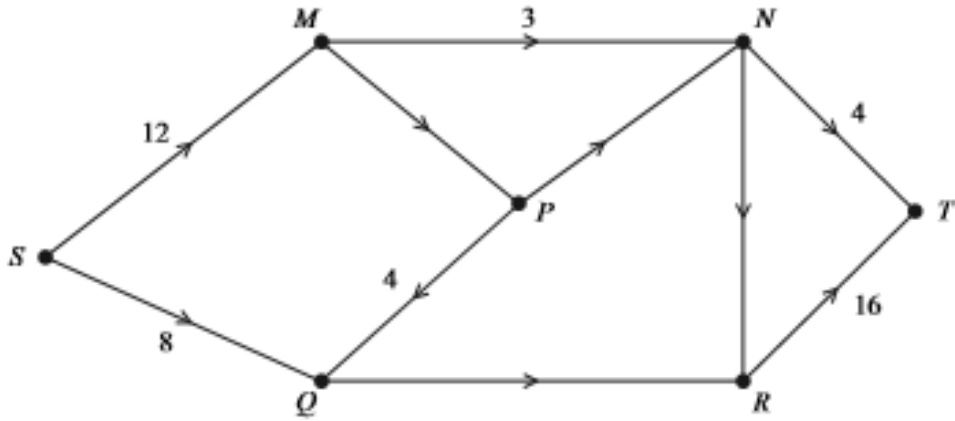
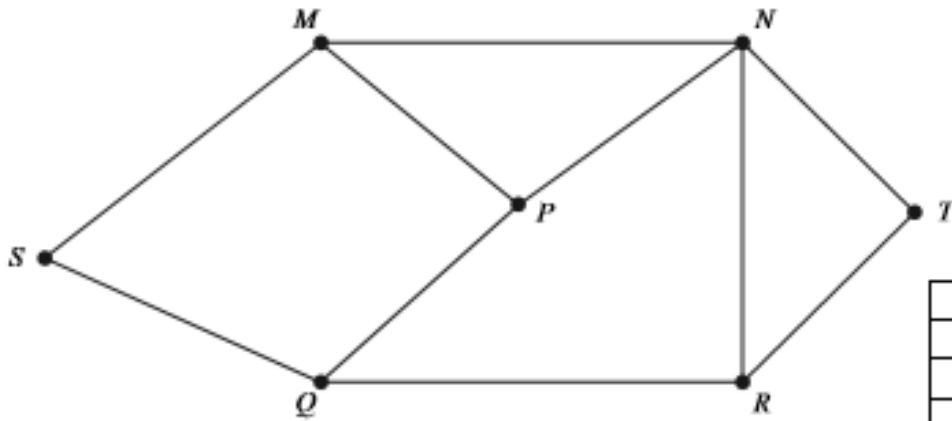


Figure 5 (for use in Question 6)



Path	Flow

Figure 6 (for use in Question 6)

