Decision 2 Dynamic Programming Questions

2 A manufacturing company is planning to build three new machines, A, B and C, at the rate of one per month. The order in which they are built is a matter of choice, but the profits will vary according to the number of workers available and the suppliers' costs. The expected profits in thousands of pounds are given in the table.

Month	Already built	Profit (in units of £1000)		
		A	***B***	***C***
1	—	52	47	48
2	***A***	—	58	54
	B	70	—	54
	C	68	63	—
3	***A* and *B***	—	—	64
	A* and *C	—	67	—
	B* and *C	69	—	—

(a) Draw a labelled network such that the most profitable order of manufacture corresponds to the longest path within that network. *(2 marks)*

(b) Use dynamic programming to determine the order of manufacture that **maximises** the total profit, and state this maximum profit. *(7 marks)*

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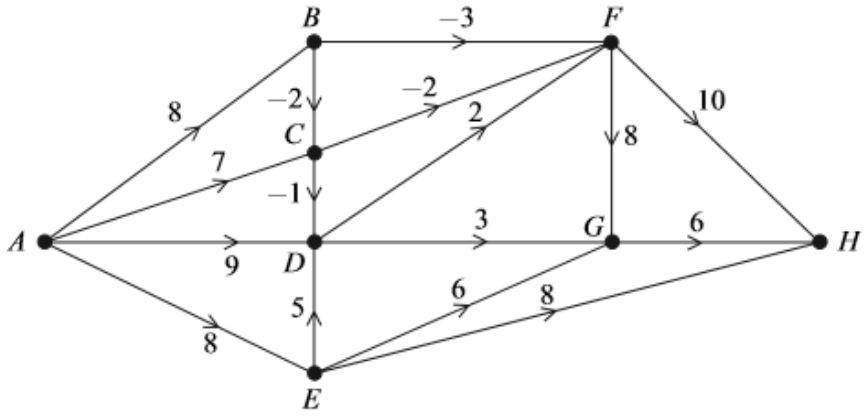
3 [**Figure 3**, printed on the insert, is provided for use in this question.]

The following network shows eight vertices. The number on each edge is the cost of travelling between the corresponding vertices. A negative number indicates a reduction by the amount shown.



(a) Use dynamic programming to find the minimum cost of travelling from A to H. You may use **Figure 3** for your working. *(6 marks)*

(b) State the minimum cost and the possible routes corresponding to this minimum cost. *(3 marks)*

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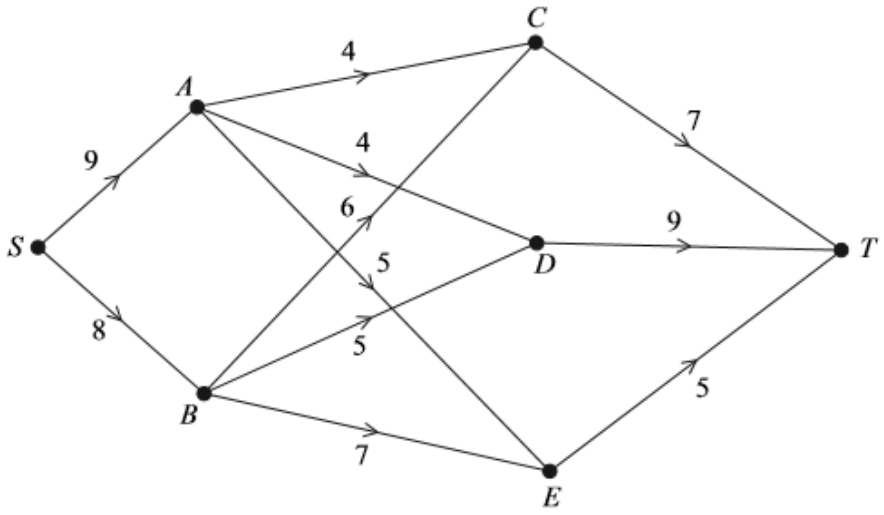
5 A three-day journey is to be made from S to T, with overnight stops at the end of the first day at either A or B and at the end of the second day at one of the locations C, D or E. The network shows the number of hours of sunshine forecast for each day of the journey.



The optimal route, known as the maximin route, is that for which the least number of hours of sunshine during a day's journey is as large as possible.

(a) Explain why the three-day route $SAET$ is better than $SACT$. *(2 marks)*

(b) Use dynamic programming to find the optimal (maximin) three-day route from S to T. *(8 marks)*

5 [**Figure 3**, printed on the insert, is provided for use in this question.]

A maker of exclusive furniture is planning to build three cabinets A, B and C at the rate of one per month. The order in which they are built is a matter of choice, but the costs will vary because of the materials available and suppliers' costs. The expected costs, in pounds, are given in the table.

Month	**Already built**	**Cost**		
		A	***B***	***C***
1	–	500	440	475
2	***A***	–	440	490
	B	510	–	500
	C	520	490	–
3	***A* and *B***	–	–	520
	A* and *C	–	500	–
	B* and *C	510	–	–

(a) Use dynamic programming, working **backwards** from month 3, to determine the order of manufacture that **minimises** the total cost. You may wish to use **Figure 3** for your working. *(6 marks)*

(b) It is discovered that the figures given were actually the profits, not the costs, for each item. Modify your solution to find the order of manufacture that **maximises** the total profit. You may wish to use the final column of **Figure 3** for your working. *(4 marks)*

Figure 3 (for use in Question 3)

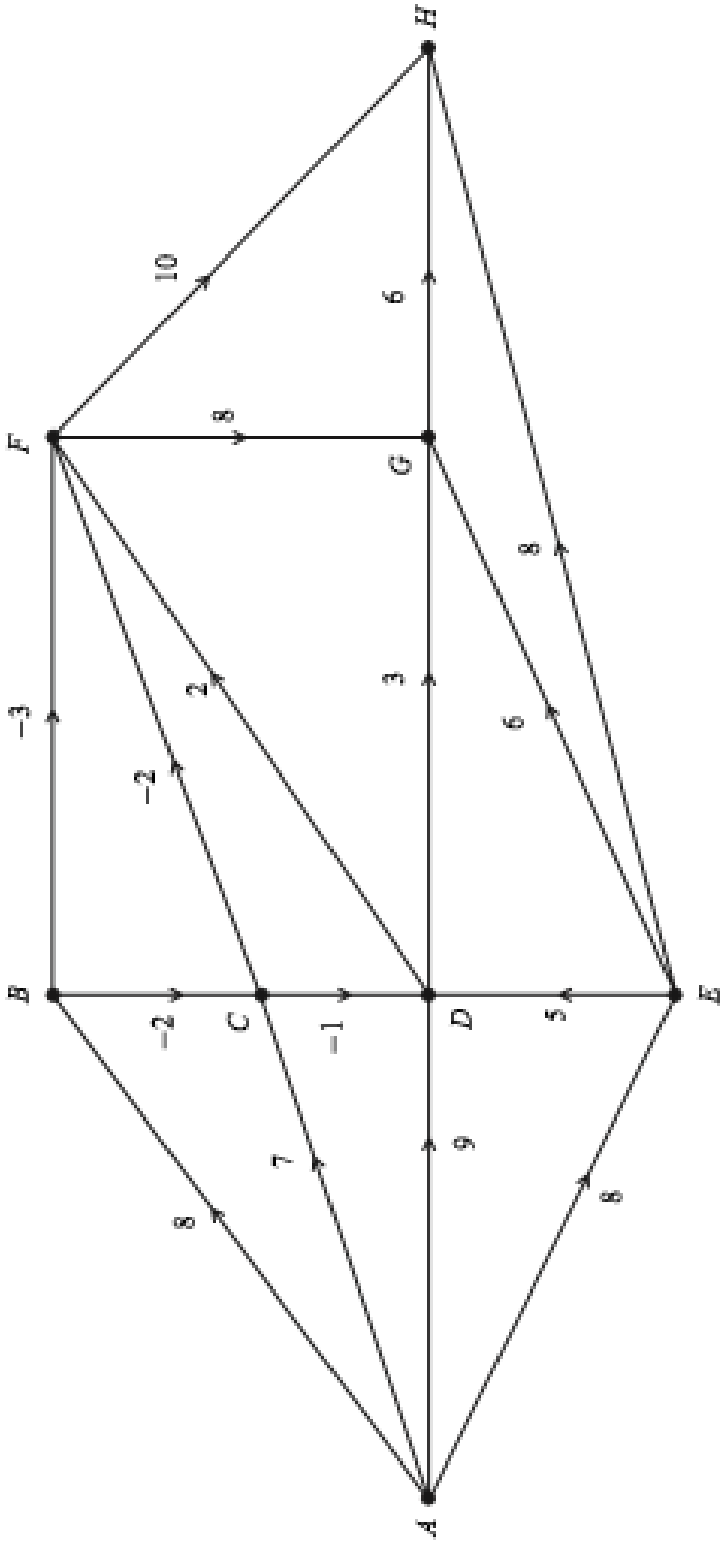


Figure 3 (for use in Question 5)

Month	**Already built**	**Machine built**	For use in part (a)		For use in part (b)
3	*A* and *B*	*C*			
	A and *C*	*B*			
	B and *C*	*A*			
2	*A*	*B*			
		C			
	B	*A*			
		C			
	C	*A*			
		B			