

## Core 4 Exponential Growth Questions

- 4 On 1 January 1900, a sculpture was valued at £80.

When the sculpture was sold on 1 January 1956, its value was £5000.

The value, £ $V$ , of the sculpture is modelled by the formula  $V = Ak^t$ , where  $t$  is the time in years since 1 January 1900 and  $A$  and  $k$  are constants.

- (a) Write down the value of  $A$ . (1 mark)
- (b) Show that  $k \approx 1.07664$ . (3 marks)
- (c) Use this model to:
- (i) show that the value of the sculpture on 1 January 2006 will be greater than £200 000; (2 marks)
- (ii) find the year in which the value of the sculpture will first exceed £800 000.
- 

- 8 A disease is spreading through a colony of rabbits. There are 5000 rabbits in the colony. At time  $t$  hours,  $x$  is the number of rabbits infected. The rate of increase of the number of rabbits infected is proportional to the product of the number of rabbits infected and the number not yet infected.

- (a) (i) Formulate a differential equation for  $\frac{dx}{dt}$  in terms of the variables  $x$  and  $t$  and a constant of proportionality  $k$ . (2 marks)
- (ii) Initially, 1000 rabbits are infected and the disease is spreading at a rate of 200 rabbits per hour. Find the value of the constant  $k$ .
- (You are **not** required to solve your differential equation.) (2 marks)
- (b) The solution of the differential equation in this model is

$$t = 4 \ln\left(\frac{4x}{5000 - x}\right)$$

- (i) Find the time after which 2500 rabbits will be infected, giving your answer in hours to one decimal place. (2 marks)
- (ii) Find, according to this model, the number of rabbits infected after 30 hours. (4 marks)
-

8 (a) (i) Solve the differential equation  $\frac{dy}{dt} = y \sin t$  to obtain  $y$  in terms of  $t$ . (4 marks)

(ii) Given that  $y = 50$  when  $t = \pi$ , show that  $y = 50e^{-(1+\cos t)}$ . (3 marks)

(b) A wave machine at a leisure pool produces waves. The height of the water,  $y$  cm, above a fixed point at time  $t$  seconds is given by the differential equation

$$\frac{dy}{dt} = y \sin t$$

(i) Given that this height is 50 cm after  $\pi$  seconds, find, to the nearest centimetre, the height of the water after 6 seconds. (2 marks)

(ii) Find  $\frac{d^2y}{dt^2}$  and hence verify that the water reaches a maximum height after  $\pi$  seconds. (4 marks)

---

4 A biologist is researching the growth of a certain species of hamster. She proposes that the length,  $x$  cm, of a hamster  $t$  days after its birth is given by

$$x = 15 - 12e^{-\frac{t}{14}}$$

(a) Use this model to find:

(i) the length of a hamster when it is born; (1 mark)

(ii) the length of a hamster after 14 days, giving your answer to three significant figures. (2 marks)

(b) (i) Show that the time for a hamster to grow to 10 cm in length is given by

$$t = 14 \ln\left(\frac{a}{b}\right), \text{ where } a \text{ and } b \text{ are integers. (3 marks)}$$

(ii) Find this time to the nearest day. (1 mark)

(c) (i) Show that

$$\frac{dx}{dt} = \frac{1}{14}(15 - x) \quad (3 \text{ marks})$$

(ii) Find the rate of growth of the hamster, in cm per day, when its length is 8 cm. (1 mark)