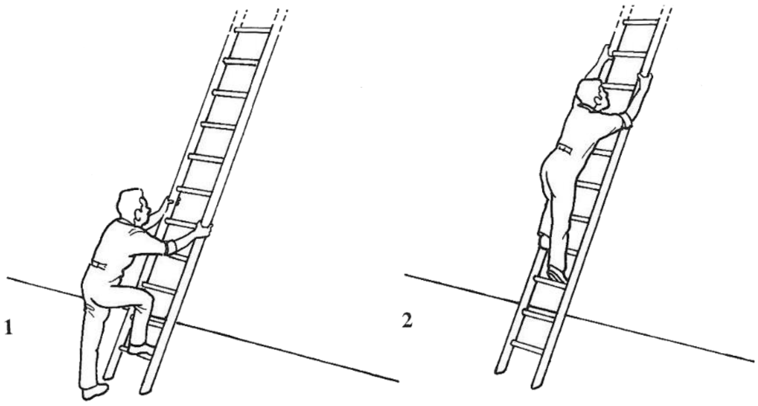
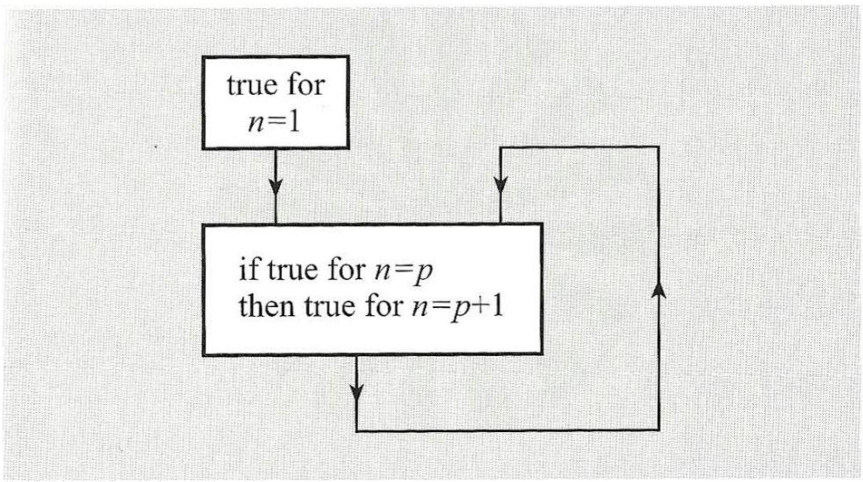
Proof By Induction

Could the boy climb to the top of the ladder?
How do you know?

Proof By Induction



	Climbing the ladder	In mathematics
1	Prove that you can reach the bottom rung of the ladder.	Prove that statement is true for $n = 1$.
2	Prove that, from any rung on the ladder, you can reach the next rung of the ladder.	Prove that for any value of n, such as k, for which the statement is true, then it will also be true for $k + 1$.
3	State that you have demonstrated that you can climb up the ladder ad infintum.	Conclude the argument.



Example 1

Prove by induction that

$$\sum_{r=1}^{r=n} r^2 = \frac{n(n+1)(2n+1)}{6}$$

1. Prove true for $n = 1$…

$$\sum_{r=1}^{r=1} r^2 = 1^2 = 1$$

$$\frac{1(1+1)(2\times 1+1)}{6} = \frac{1\times 2\times 3}{6} = 1$$

2. Assume true for $n = k$, prove true for $n = k+1$…

$\sum_{r=1}^{r=k+1} r^2$	
$= \left(\sum_{r=1}^{r=k} k^2\right) + (k+1)^2$	$= \sum_{r=1}^{r=k+1} r^2$
$= \left(\frac{k(k+1)(2k+1)}{6}\right) + (k+1)^2$	$= \frac{(k+1)\big((k+1)+1\big)(2(k+1)+1)}{6}$
Prove that these two equations are the same	

3. Conclude…

If the result is true for $n = k$, then it is true for $n = k+1$.
As it is true for $n = 1$, then it is true for all $n \geq 1$ by induction.

Example 2

Prove by induction that

$$\sum_{r=1}^{r=n} r^2(r+1) = \frac{n(n+1)(n+2)(3n+1)}{12}$$

1. Prove true for $n = 1$...

$$\sum_{r=1}^{r=1} r^2(r+1) = 1^2 + (1+1) = 2$$

$$\frac{1(1+1)(1+2)(3\times 1+1)}{12} = \frac{1\times 2\times 3\times 4}{12} = 2$$

2. Assume true for $n = k$, prove true for $n = k+1$...

$\sum_{r=1}^{r=k+1} r^2\,(r+1)$	
$= \left(\sum_{r=1}^{r=k} k^2(k+1)\right) + (k+1)^2\big((k+1)+1\big)$	$= \sum_{r=1}^{r=k+1} r^2\,(r+1)$
$= \left(\frac{k(k+1)(k+2)(3k+1)}{12}\right) + (k+1)^2\big((k+1)+1\big)$	$= \frac{(k+1)\big((k+1)+1\big)\big((k+1)+2\big)\big(3(k+1)+1\big)}{12}$
Prove that these two equations are the same	

3. Conclude...

If the result is true for $n = k$, then it is true for $n = k+1$.
As it is true for $n = 1$, then it is true for all $n \geq 1$ by induction.

Example 3

$$u_{n+1} = 4u_n - 3 \qquad u_1 = 2$$

Prove by induction that

$$u_n = 4^{n-1} + 1$$

1. Prove true for $n = 1$…

$$u_1 = 2$$

$$u_1 = 4^{1-1} + 1 = 4^0 + 1 = 2$$

2. Assume true for $n = k$, prove true for $n = k + 1$…

$u_{k+1} = 4^{(k+1)-1} + 1$
$u_{k+1} = 4u_k - 3$ $= 4(4^{k-1} + 1) - 3$ $= 4 \times 4^{k-1} + 4 - 3$ $= 4^k + 1$

3. Conclude…

If the result is true for $n = k$, then it is true for $n = k + 1$.
As it is true for $n = 1$, then it is true for all $n \geq 1$ by induction.