A False Proof

Appendix 7. Straying into Absurdity

The following is a classic demonstration of how easy it is to start off with a very simple statement and then within a few apparently straightforward and logical steps show that $2 = 1$.

First, let us begin with the innocuous statement

$$a = b.$$

Then multiply both sides by a, giving

$$a^2 = ab.$$

Then add $a^2 - 2ab$ to both sides:

$$a^2 + a^2 - 2ab = ab + a^2 - 2ab.$$

This can be simplified to

$$2(a^2 - ab) = a^2 - ab.$$

Finally, divide both sides by $a^2 - ab$, and we get

$$2 = 1.$$

The original statement appears to be, and is, completely harmless, but somewhere in the step-by-step manipulation of the equation there was a subtle but disastrous error which leads to the contradiction in the final statement.

In fact, the fatal mistake appears in the last step in which both sides are divided by $a^2 - ab$. We know from the original statement that $a = b$, and therefore dividing by $a^2 - ab$ is equivalent to dividing by zero.

Dividing anything by zero is a risky step because zero will go into any finite quantity an infinite number of times. By creating infinity on both sides we have effectively torn apart both halves of the equation and allowed a contradiction to creep into the argument.

This subtle error is typical of the sort of blunder which caught out many of the entrants for the Wolfskehl Prize.