

Perms & Combs

- How many arrangements of the letters A B C D E ?

$$5! = 5 \times 4 \times 3 \times 2 \times 1 = 120$$

$$n!$$

- How many arrangements of the letters A A B B B C ?

$$\frac{6!}{2!3!} = \frac{720}{12} = 60$$

$$\frac{n!}{p!q!r!} \quad \text{where } p, q, r \text{ are the number of repeat letters}$$

- How many arrangements of A B C D E so that A & B are together?
(Treating A & B as one unit with 2! arrangements of their subset)

$$4!2! = 48$$

Permutations – order matters, so EACH possible selection will be rearranged in all possible orders.

- How many arrangements of 4 letters from 7 different letters?

$${}^7P_4 = 840$$

Combinations – order doesn't matter, so each possible selection counts only once.

- How many ways to choose 11 players from a squad of 16?

$${}^{16}C_{11} = 4368$$

- Number of ways to choose 11 from 16 is same as number of ways to choose 5 from 16

$${}^{16}C_{11} = {}^{16}C_5$$

$${}^nC_r = {}^nC_{(n-r)}$$

- 2 Sets - How many ways to choose 2 from 10 **and** 3 from another set of 8? (The 'and \Rightarrow multiply' rule)

$${}^{10}C_2 \times {}^8C_3 = 45 \times 56 = 2520$$

- Multiple options – How many ways to choose 2 from 10 and 3 from another set of 8, **OR**, 4 from 10 and 1 from 8? (The 'or \Rightarrow add' rule)

$$({}^{10}C_2 \times {}^8C_3) + ({}^{10}C_4 \times {}^8C_1) = (45 \times 56) + (210 \times 8) = 2520 + 1680 = 4200$$

- Probabilities based on selections

$$\frac{\text{Number of selections satisfying criteria}}{\text{Total number of possible selections}}$$

e.g. 10 people in a room. What is the probability that A and B sit next to each other?

Number of selections with A,B next to each other: $9! \times 2! = 725760$

Total number of ways 10 people can sit: $10! = 3628800$

Probability is therefore $725760/3628800$

- "What is the *permutation* for the safe?"

Permutations

$${}^n P_r$$

Order Matters

Combinations

$${}^n C_r$$

Order doesn't matter