

## Parametric Equations

Be able to find a Cartesian equation / eliminate the third variable (or find an implicit equation).

The smaller part

Be able to differentiate,  
find  $\frac{dy}{dx}$  (usually in terms of  $t$ ).

The bigger part

Simpler examples such as...

$$x = t + 1$$

$$y = t^2$$

Harder examples such as...

$$x = 2\sin t$$

$$y = 5 - 4\cos t$$

$$x = 3t + \frac{1}{t}$$

$$y = 3t - \frac{1}{t}$$

Note that...

$$x^2 - y^2 = -8$$

is an implicit function

$$y = \sqrt{x^2 + 8}$$

is an explicit function

$$\frac{dy}{dx} = \frac{dy}{dt} \times \frac{dt}{dx}$$

## Rearrangements of Examples

The Simpler Example

$$x = t + 1$$

$$y = t^2$$

$$t = (x - 1) \Rightarrow y = (x - 1)^2$$

Harder Example 1:

$$\begin{aligned}x &= 2\sin t \\ \Rightarrow \frac{x}{2} &= \sin t \\ \Rightarrow \frac{x^2}{4} &= \sin^2 t\end{aligned}$$

$$\begin{aligned}y &= 5 - 4\cos t \\ \Rightarrow y - 5 &= -4\cos t \\ \Rightarrow \frac{5 - y}{4} &= \cos t \\ \Rightarrow \frac{(5 - y)^2}{16} &= \cos^2 t\end{aligned}$$

$$\frac{x^2}{4} + \frac{(5 - y)^2}{16} = 1$$

Harder Example 2:

$$x = 3t + \frac{1}{t}$$

$$y = 3t - \frac{1}{t}$$

$$\begin{aligned}x + y &= 6t \\ x - y &= \frac{2}{t}\end{aligned}$$

$$(x + y)(x - y) = 6t \times \frac{2}{t} = 12$$

$$(x + y)(x - y) = 12$$

## Differentiation of Examples

E.g. 1

$$x = 2t + 1$$

$$y = t^2$$

$$\frac{dx}{dt} = 2$$

$$\frac{dy}{dt} = 2t$$

so...

$$\frac{dt}{dx} = \frac{1}{2}$$

And therefore...

$$\frac{dy}{dx} = \frac{dy}{dt} \times \frac{dt}{dx} = 2t \times \frac{1}{2} = t$$

**E.g.2**

$$x = 2\sin t$$

$$y = 5 - 4\cos t$$

$$\frac{dx}{dt} = 2\cos t$$

$$\frac{dy}{dt} = 4\sin t$$

so...

$$\frac{dt}{dx} = \frac{1}{2\cos t}$$

And therefore...

$$\frac{dy}{dx} = \frac{dy}{dt} \times \frac{dt}{dx} = 4\sin t \times \frac{1}{2\cos t} = 2\tan t$$

**E.g.3**

$$x = 3t + \frac{1}{t}$$

$$y = 3t - \frac{1}{t}$$

$$\frac{dx}{dt} = 3 - \frac{1}{t^2}$$

$$\frac{dy}{dt} = 3 + \frac{1}{t^2}$$

$$\frac{dt}{dx} = \frac{1}{3 - \frac{1}{t^2}}$$

And therefore...

$$\frac{dy}{dx} = \frac{dy}{dt} \times \frac{dt}{dx} = \left(3 + \frac{1}{t^2}\right) \times \left(\frac{1}{3 - \frac{1}{t^2}}\right) = \frac{3 + \frac{1}{t^2}}{3 - \frac{1}{t^2}} = \frac{3t^2 + 1}{3t^2 - 1}$$

Which rearranges to...

$$\frac{3 + \frac{1}{t^2}}{3 - \frac{1}{t^2}} \times \frac{t^2}{t^2} = \frac{3t^2 + 1}{3t^2 - 1}$$