

Methods for Integration

What do the questions for each method look like?

Inverse Chain Rule	Partial Fractions	Logarithms	Integration by Parts	Integration by Substitution
$\int 4x^2(x^3 - 3)^5 dx$	$\int \frac{10x^2 + 8}{(x + 1)(5x - 1)} dx$	$\int_2^3 \frac{2x^2 - 1}{4x^3 - 6x + 1} dx$	$\int x^2 \sin 4x dx$	$\int \frac{x^2}{2x - 1} dx$

What should I look for to recognise each method?

Order of function outside the bracket is one less than order of function inside .	Factors in the denominator	Numerator is (a multiple of) the differential of the denominator	Product of two functions	A function within a function
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What else might the questions look like?

$\int \frac{x}{\sqrt{x^2 + 3}} dx$	$\int \frac{6x - 5}{4x^2 - 25} dx$	$\int \frac{2x^3 + 1}{x^4 + 2x} dx$	$\int \ln x dx$	$\int x \sqrt{2x + 1} dx$
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Anything else I should know about this method?

<p>“Integrate whole thing then <i>divide</i> by the differential of the function inside”</p>	<p>You’ll need to be able to integrate functions such as...</p> $\int \frac{1}{2x + 1} dx = \frac{1}{2} \ln(2x + 1)$	<p>Remember this...</p> $\int \frac{f'(x)}{f(x)} dx = \ln f(x)$	$uv - \int v du dx$ <p>Start with...</p> <p>$u = \dots$ and $dv = \dots$</p> <p>Aim to make the function inside the integral simpler</p>	<p>Start with...</p> $\frac{du}{dx} = \dots$ $dx = \dots du$ <p>$x = \dots$</p>
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Methods for Integration

Practice Questions

Inverse Chain Rule	Partial Fractions	Logarithms	Integration by Parts	Integration by Substitution
Integrate these:	Integrate these:	Integrate these:	Integrate these:	Integrate these:
1. $\int 2x(4 - 3x^2)^5 dx$	1. $\int \frac{5x-6}{x(x-3)} dx$	1. $\int_4^6 \frac{x-2}{2x^2-8x+3} dx$	1. $\int e^{4x}(2x + 1) dx$	1. $\int \frac{1+\ln x}{x} dx$
2. $\int \frac{1}{\sqrt{2x-1}} dx$	2. $\int \frac{2}{x^2-1} dx$	2. $\int \frac{x^2}{x^3+3} dx$	2. $\int x^{-2} \ln x dx$	2. $\int_0^1 4x \ln(2x + 1) dx$
3. $\int x^3 \sqrt{x^4 - 1} dx$	3. $\int \frac{3x-5}{(x+3)(2x-1)} dx$	3. $\int \frac{2e^x}{e^x+3} dx$	3. $\int \ln(4x - 3) dx$	3. $\int \frac{4x}{4x-3} dx$
4. $\int (4x) \sqrt[3]{2 - 3x^2} dx$	4. $\int \frac{28+4x^2}{(3x+1)(5-x)^2} dx$	4. $\int_2^3 \frac{2x^2-1}{4x^3-6x+1} dx$	4. $\int x^2 \sin 4x dx$	4. $\int \frac{1}{(1+2\tan x)^2 \cos^2 x} dx$
5. $\int \frac{25x^4}{(3-x^5)^2} dx$	5. $\int \frac{3x-5}{x-3} dx$	5. $\int \frac{2x^3+1}{x^4+2x} dx$	5. $\int_0^4 x^2 e^{-\frac{x}{4}} dx$	5. $\int x\sqrt{3x+1} dx$
6. $\int x^{\frac{1}{3}} (x^{\frac{4}{3}} - 2)^2 dx$	6. $\int \frac{9x^2-6x+5}{(3x-1)(x-1)} dx$	6. $\int \frac{3x^3+x}{(3x^2+1)^2} dx$	6. $\int x e^{-2x} dx$	6. $\int_0^1 \frac{x^7}{(x^4+2)^2} dx$

You choose the method

1. $\int \frac{1}{1+x} dx$	2. $\int \frac{x}{1+x} dx$	3. $\int \frac{1}{1+x^2} dx$	4. $\int \frac{1}{1-x^2} dx$	5. $\int \frac{1}{(1+x)^2} dx$	6. $\int \frac{1}{\sqrt{1+x}} dx$
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Methods for Integration - Answers

Practice Questions

Inverse Chain Rule	Partial Fractions	Logarithms	Integration by Parts	Integration by Substitution
<p>Integrate these:</p> <ol style="list-style-type: none"> $\frac{-(4-3x^2)^6}{18}$ $\sqrt{2x-1}$ $\frac{\sqrt{(x^4-1)^3}}{6}$ $\frac{-\sqrt[3]{(2-3x^2)^4}}{2}$ $\frac{5}{3-x^5}$ $\frac{\left(x^{\frac{4}{3}}-2\right)^3}{4}$ 	<p>Integrate these:</p> <ol style="list-style-type: none"> $2\ln x + 3\ln(x-3) + c$ $\ln(x-1) - \ln(x+1) + c$ $2\ln(x+3) - \frac{1}{2}\ln(2x-1) + c$ $\frac{1}{3}\ln(3x+1) + \ln(5-x) + \frac{8}{5-x} + c$ $3x + 4\ln(x-3) + c$ $-2\ln(3x-1) + 4\ln(x-1) + c$ 	<p>Integrate these:</p> <ol style="list-style-type: none"> $\frac{1}{4}(\ln 27 - \ln 3)$ $\frac{1}{3}\ln(x^3 + 3) + c$ $2\ln(e^x + 3) + c$ $\frac{1}{6}\ln \frac{91}{21}$ $\frac{1}{2}\ln(x^4 + 2x) + c$ $\frac{1}{12}(3x^2 + 1)^2 + c$ 	<p>Integrate these:</p> <ol style="list-style-type: none"> $\frac{e^{4x}(2x+1)}{4} - \frac{e^{4x}}{8} + c$ $-\frac{1}{x}\ln x - \frac{1}{x} + c$ $x\ln(4x-3) - \frac{1}{4}[(4x-3) + 3\ln(4x-3)] + c$ $\frac{-x^2\cos 4x}{4} + \frac{x\sin 4x}{8} + \frac{\cos 4x}{32} + c$ $128 - \frac{320}{e}$ $\frac{-xe^{-2x}}{2} - \frac{e^{-2x}}{4} + c$ 	<p>Integrate these:</p> <ol style="list-style-type: none"> $\frac{(1+\ln x)^2}{2} + c$ $\frac{3}{2}\ln 3$ $\frac{1}{4}[(4x-3) + 3\ln(4x-3)] + c$ $\frac{-1}{2(1+2\tan x)} + c$ $\frac{2}{45}(3x-1)^{\frac{5}{2}} + \frac{2}{27}(3x-1)^{\frac{3}{2}} + c$ $\frac{1}{4}\ln\left(\frac{3}{2}\right) - \frac{1}{12} + c$

You choose the method

1. $\ln(1+x)$	2. $x - \ln(1+x)$	3. $\tan^{-1}x$	4. $\frac{1}{2}\ln\left(\frac{1+x}{1-x}\right)$	5. $\frac{1}{1+x}$	6. $2\sqrt{1+x}$
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