Maths Tricks - Links and Notes

General Notes:

- http://mathsbusking.com has more great maths tricks.
- http://www.amazon.co.uk/Think-Number-Johnny-Ball/dp/1405358025/ref=sr_1_2?s=books&ie=UTF8&qid=1360008145&sr=1-2 is a great book by Johnny Ball about the history of maths and many mathematical tricks and quirks.

Question Specific:

1) A great book on this problem and others http://www.amazon.co.uk/1089-All-That-Journey-Mathematics/dp/0199590028/ref=sr_1_1?ie=UTF8&qid=1360006606&sr=8-1

2) The person who knows is in the middle on the right hand side of the wall. The key is "*after a minute or two*"; if the hats of the middle people were the same colour then the person at the back would know the colour of their hat. If they haven't spoken then the person in the middle will realize that the middle hats are different, and therefore that theirs is the opposite colour to the one in front.

3) Answer is given by $\dfrac{8}{3-\frac{8}{3}}$.

4) (no notes)

5) The error is made in dividing by a-b since this is zero and dividing by zero is undefined (infinity / impossible).

6) The key here is not in *saving time* but rather *not wasting time*. Therefore, the 5 and 10 minute people must go together. Solution is; 1 & 2 cross (total 2), 1 returns (total 3), 5 & 10 cross (total 13), 2 returns (total 15), 2 & 1 cross again (total 17).

7) See http://mathsbusking.com/shows/cubic_root_whiz/ for more on this problem.

8) (no notes)

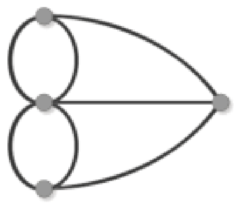
9) (no notes)

10) Large version of the cards http://www.colmanweb.co.uk/Assets/Resources/NumberSystems/Binarycards.ppt.
A lesson based on this task http://www.colmanweb.co.uk/Assets/Resources/NumberSystems/Binary.doc.

11) Answer is given by $\dfrac{6}{1-\frac{3}{4}}$.

12)The trick here is in the misleading phrase "this totals £29 instead of £30". The £2 should be subtracted from £27 instead of adding; £27 - £2 = £25. Alternatively, £25 + £2 = £27 (cost of meal and tip).

13)Reduce this problem down to this network diagram (graph):



and notice that the order of each node is odd (order 3, 3, 3, 5). Any graph with more than 2 nodes of odd order is non-eulerian and therefore non-traversible. More on this at http://nrich.maths.org/2484 and http://en.wikipedia.org/wiki/Eulerian_path.

14)Since each domino will cover one white square and one black square for every white square covered there must be one black square. Since two black squares are missing two white squares must remain uncovered and hence, the board cannot be covered. See http://en.wikipedia.org/wiki/Mutilated_chessboard_problem.

15)See http://mathsbusking.com/shows/divine_reminder/ for more on this problem.

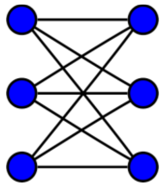
16)Commonly known as the Monty Hall problem, see http://www.youtube.com/watch?v=mhlc7peGlGg for a good explanation. This problem even has its own website; http://montyhallproblem.com! An alternate explanation for students who really do insist is to extend the problem to one of 100 doors containing 99 goats and 1 star prize. With the contestant having chosen a door, the host would reveal 98 goat-doors leaving just one other door. The choice becomes obvious.

17)(no notes)

18)(no notes)

19)(no notes)

20)The graph produced from any solution to this problem will be a bipartite graph $K_{3,3}$ for which Euler's relationship, $R + N = A + 2$, for planar graphs does not hold.



See the following for more explanation:
http://en.wikipedia.org/wiki/Water,_gas,_and_electricity
http://www.youtube.com/watch?v=hjAP8Fy5WhE

21)More notes on this activity from Nrich at http://nrich.maths.org/745/solution.

22) Labelling the width and height of the rectangle as a and b, we can then use information given to create two equations:

$$a+b=14$$
$$a^2+b^2=144$$

The obvious solution is then to solve by simultaneous equations (where a and b are surds) but there is a far more elegant solution given by considering the following:

$$(a+b)^2=a^2+2ab+b^2$$
$$\Rightarrow 196=2ab+144$$
$$\Rightarrow 52=2ab$$
$$\Rightarrow 26=ab$$

23) On each go, the probability of winning with the coins is $\frac{1}{1024}$. The expected pay-out per go is therefore $\frac{1}{1024}\times 400=39\text{p}$. On each go, the probability of winning with the spinners is $\frac{1}{3125}$. The expected pay-out per go is therefore $\frac{1}{3125}\times 1000=32\text{p}$. Therefore players should choose the coins. (The trivial case is not to play at all since both games cost £1 to play and represent an overall loss in terms of pay-outs. But then this is neither fun nor in the spirit of supporting the school and its summer fayre.)

With the coins, the school would expect to pay-out every 1,024 goes and could expect that this payout occurs on the median 512$^{\text{th}}$ attempt. By this time they would have accumulated enough money to pay for the prize. The breakeven point is the 400$^{\text{th}}$ player. The probability of the school reaching this point follows a Poisson distribution

$$X \sim P(\tfrac{1023}{1024})$$
$$P(X>400)=(\tfrac{1023}{1024})^{400}=0.676$$

With the spinners, the school would expect to pay-out every 3,125 goes and could expect that this payout occurs on the median 1,563$^{\text{rd}}$ attempt. By this time they would have accumulated enough money to pay for the prize. The breakeven point is the 1000$^{\text{th}}$ player. The probability of the school reaching this point follows a Poisson distribution

$$X \sim P(\tfrac{3124}{3125})$$
$$P(X>1000)=(\tfrac{3124}{3125})^{1000}=0.726$$

How much money can the school expect to make through each game in the long term? How much money can the school expect to make through the stall (both games) in the long term?

Possible extensions include:

a) What if both games were changed to 'get all showing the same result'? Implications for players, implications for the school?
b) What about another game where contestants can flip a coin three times and, if they get three heads they win £100 but if they get three tails they pay £10. Would students choose to play this game or not? Is this game a good idea for the school to offer?

See also http://www.youtube.com/watch?v=rwvIGNXY21Y

24) By representing the starting numbers algebraically the pattern continues as:

Row Number	a terms		b terms
1			a
2			b
3	a	+	b
4	a	+	2b
5	2a	+	3b
6	3a	+	5b
7	5a	+	8b
8	8a	+	13b
9	13a	+	21b
10	21a	+	34b
Total	55a	+	88b

The following factorisation can be made:

$$55a + 88b = 11(5a + 8b)$$

where:

$5a + 8b$ = the 7th row of the table

25) A great Numberphile explanation at http://www.youtube.com/watch?v=dHzUQnRjbuM.

26)The answer is always 37 because:

$$\frac{100x + 10x + x}{3x} = \frac{111x}{3x} = \frac{111}{3} = 37$$

27)The most common wrong answer is 19.2cm, which is achieved by subtracting the thicknesses of the two outer covers from the width of both books. The correct answer is 8mm and this is because the letter A occurs at the beginning of the first book which is to its right on the shelf whilst the letter Z occurs at the end of the second book which is to its left on the shelf.

A simple yet powerful demonstration is to give someone a book, ask them to show you where A occurs and then ask them to put the book on the shelf.

28) $3^{444} + 4^{333}$ is a multiple of 5. Since all multiples of 5 end in 5 or 0, we just need to know the unit digit $3^{444} + 4^{333}$. The pattern of the units digit of 3^1, 3^2, 3^3, 3^4, 3^5 etc repeats as $3^1 \rightarrow 3$ $3^2 \rightarrow 9$ $3^3 \rightarrow 7$ $3^4 \rightarrow 1$ $3^5 \rightarrow 3$, 9, 7, 1 etc. Specifically, $3^{4n+1} \rightarrow 3$ $3^{4n+2} \rightarrow 9$ $3^{4n+3} \rightarrow 7$ $3^{4n+4} \rightarrow 1$ so the units digit of 3^{444} is 1. Similarly, the units digit of 4^{333} is 4. 4+1=5, therefore $3^{444} + 4^{333}$ is a multiple of 5.

29) Camels

30) Coin rolling

31) If restricted to two numbers (and hence calculating the maximum area) then the answer is $5 \times 5 = 25$. If restricted to three numbers (max volume) then we have $\left(\frac{10}{3}\right)^3 = \frac{1000}{27} = 37.037$. Beyond this $3^2 \times 2^2 = 36$ or $2^5 = 32$ are both interesting solutions which ultimately lead to $e^{\left(\frac{10}{e}\right)}$ and $\left(\frac{10}{e}\right)^e = 39.4$. Graphs of these, such as $y = e^{\left(\frac{10}{e}\right)}$ and $y = \left(\frac{10}{e}\right)^e$ are both interesting, as is the three dimensional $z = x \times y \times (10 - x - y)$.