

How to get marks for Integration by Substitution questions

1. $\frac{du}{dx} =$
2. Correct integral all in terms of u (i.e. no x 's) and including the du
3. Correct simplification of the integral, ready for integrating
4. Correct integration
5. Correct conversion of limits or re-substitution for x 's
6. Correct final answer in required format

(Note that there may be more marks available if question is more complicated)

How to get marks for Integration by Substitution Questions

Question & substitution	$\int_1^3 \frac{x^2}{2x-1} dx$ $u = 2x - 1$	$\int_0^1 x\sqrt{3x+1} dx$ $u = 3x + 1$	$\int_{\sqrt{2}}^2 \frac{1}{x^2\sqrt{x^2-1}} dx$ $x = \operatorname{cosec}\theta$	$\int_0^1 \frac{x^7}{(x^4+2)^2} dx$ $u = x^4 + 2$	$\int_1^4 \frac{1}{x+\sqrt{x}} dx$ $u = \sqrt{x}$
$\frac{du}{dx} =$			$\frac{dx}{d\theta} =$		
Correct integral all in terms of u (i.e. no x's) and including the du					
Correct simplification of the integral, ready for integrating					
Correct integration					
Correct conversion of limits or re-substitution for x's					
Correct final answer in required format					
Exam Paper Ref.	Jan 2010	Jan 2011	Jan 2012	June 2012	June 2013

How to get marks for Integration by Substitution Questions - Answers

Question & substitution	$\int_1^3 \frac{x^2}{2x-1} dx$ $u = 2x - 1$	$\int_0^1 x\sqrt{3x+1} dx$ $u = 3x + 1$	$\int_{\sqrt{2}}^2 \frac{1}{x^2\sqrt{x^2-1}} dx$ $x = \operatorname{cosec}\theta$	$\int_0^1 \frac{x^7}{(x^4+2)^2} dx$ $u = x^4 + 2$	$\int_1^4 \frac{1}{x+\sqrt{x}} dx$ $u = \sqrt{x}$																														
$\frac{du}{dx} =$	$= 2$	$= 3$	$\frac{dx}{d\theta} = -\operatorname{cosec}\theta\cot\theta$ $= \frac{-1}{\sin\theta} \cdot \frac{\cos\theta}{\sin\theta}$	$= 4x^3$	$= \frac{1}{2}u^{-\frac{1}{2}} = \frac{1}{2\sqrt{x}}$																														
Correct integral all in terms of u (i.e. no x's) and including the du	$\int \left(\frac{u+1}{2}\right)^2 \left(\frac{1}{u}\right) \frac{du}{2}$	$\int \frac{u+1}{2} \sqrt{u} \frac{du}{3}$	$\int \frac{1}{\operatorname{cosec}^2\theta\sqrt{\operatorname{cosec}^2\theta-1}} \cdot \frac{-\cos\theta}{\sin^2\theta} d\theta$	$\int \frac{u-2}{u^2} du$	$\int \frac{1}{u^2+u} \cdot 2u du$																														
Correct simplification of the integral, ready for integrating	$\frac{1}{8} \int u + 2 + \frac{1}{u} du$	$\frac{1}{9} \int u^{\frac{3}{2}} - u^{\frac{1}{2}} du$	$-\int \sin\theta d\theta$	$\int u^{-1} - 2u^{-2} du$	$2 \int \frac{1}{u+1} du$																														
Correct integration	$\frac{1}{8} \left[\frac{u^2}{2} + 2u + \ln u \right]$	$\frac{1}{9} \left[\frac{2u^{\frac{5}{2}}}{5} - \frac{2u^{\frac{3}{2}}}{3} \right]$	$\cos\theta$	$\left[\ln u + \frac{2}{u} \right]$	$2[\ln(u+1)]$																														
Correct conversion of limits or re-substitution for x's	<table border="1" style="display: inline-table; vertical-align: middle;"><tr><td style="border-right: 1px solid black; padding: 2px 5px;">x</td><td style="padding: 2px 5px;">u</td></tr><tr><td style="border-right: 1px solid black; padding: 2px 5px;">3</td><td style="padding: 2px 5px;">5</td></tr><tr><td style="border-right: 1px solid black; padding: 2px 5px;">1</td><td style="padding: 2px 5px;">1</td></tr></table>	x	u	3	5	1	1	<table border="1" style="display: inline-table; vertical-align: middle;"><tr><td style="border-right: 1px solid black; padding: 2px 5px;">x</td><td style="padding: 2px 5px;">u</td></tr><tr><td style="border-right: 1px solid black; padding: 2px 5px;">1</td><td style="padding: 2px 5px;">4</td></tr><tr><td style="border-right: 1px solid black; padding: 2px 5px;">0</td><td style="padding: 2px 5px;">1</td></tr></table>	x	u	1	4	0	1	<table border="1" style="display: inline-table; vertical-align: middle;"><tr><td style="border-right: 1px solid black; padding: 2px 5px;">θ</td><td style="padding: 2px 5px;">x</td></tr><tr><td style="border-right: 1px solid black; padding: 2px 5px;">2</td><td style="padding: 2px 5px;">$\frac{\pi}{6}$</td></tr><tr><td style="border-right: 1px solid black; padding: 2px 5px;">$\sqrt{2}$</td><td style="padding: 2px 5px;">$\frac{\pi}{4}$</td></tr></table>	θ	x	2	$\frac{\pi}{6}$	$\sqrt{2}$	$\frac{\pi}{4}$	<table border="1" style="display: inline-table; vertical-align: middle;"><tr><td style="border-right: 1px solid black; padding: 2px 5px;">x</td><td style="padding: 2px 5px;">U</td></tr><tr><td style="border-right: 1px solid black; padding: 2px 5px;">1</td><td style="padding: 2px 5px;">3</td></tr><tr><td style="border-right: 1px solid black; padding: 2px 5px;">0</td><td style="padding: 2px 5px;">2</td></tr></table>	x	U	1	3	0	2	<table border="1" style="display: inline-table; vertical-align: middle;"><tr><td style="border-right: 1px solid black; padding: 2px 5px;">x</td><td style="padding: 2px 5px;">u</td></tr><tr><td style="border-right: 1px solid black; padding: 2px 5px;">4</td><td style="padding: 2px 5px;">2</td></tr><tr><td style="border-right: 1px solid black; padding: 2px 5px;">1</td><td style="padding: 2px 5px;">1</td></tr></table>	x	u	4	2	1	1
x	u																																		
3	5																																		
1	1																																		
x	u																																		
1	4																																		
0	1																																		
θ	x																																		
2	$\frac{\pi}{6}$																																		
$\sqrt{2}$	$\frac{\pi}{4}$																																		
x	U																																		
1	3																																		
0	2																																		
x	u																																		
4	2																																		
1	1																																		
Correct final answer in required format	$\frac{5}{2} + \frac{\ln 5}{8}$	$\frac{116}{135}$	$\frac{\sqrt{3}-\sqrt{2}}{2}$	$\ln\left(\frac{3}{2}\right) - \frac{1}{3}$	$2\ln\left(\frac{3}{2}\right)$																														
Exam Paper Ref.	Jan 2010	Jan 2011	Jan 2012	June 2012	June 2013																														