Mappings & Functions

$y = 3x + 1$ $\quad f(x) = 3x+1$ $\quad x \rightarrow 3x+1$

Domain	Starting value	"x"
Range or Co-Domain	Finishing value	"$f(x)$" or "y"

Types of Mappings

Type	Example	Proof
One to one	$f(x) = 3x+1$	$f(1) = 4,\ f(-1) = -2$ (Not the same!)
Many to one	$f(x) = \sin x$	$f(30) = 0.5,\ f(150) = 0.5$ …
Many to many	$\frac{x^2}{a} + \frac{y^2}{b} = r^2$	$y = \pm\sqrt{b\left(r^2 - \frac{x^2}{a}\right)}$

The following is mapping but is not a function…

One to many	$f(x) = \sin^{-1} x$	$f(0.5) = 30,\ f(0.5) = 150$ …

To clarify…

Type	Mapping	Function
One to one	Yes	Yes
Many to one	Yes	Yes
Many to many	Yes	Yes
One to many	Yes	No

The difference between inverse-sin and arc-sin etc.

Name	Function	Domain
Inverse-sin	$sin^{-1}x$	$-\infty \leq x \leq \infty$
Arc-sin	$sin^{-1}x$	$-\frac{\pi}{2} \leq x \leq \frac{\pi}{2}$

- To change a 'many to one' function to a 'one to one' function we restrict the domain:

$f(x) = x^2$	$x \geq 0$
$f(x) = \sin x$	$-90 \leq x \leq +90$
$f(x) = \cos x$	$0 \leq x \leq 180$
$f(x) = \tan x$	$-90 \leq x \leq +90$

Composite Functions

Given:

$$f(x) = x + 3 \quad \text{and} \quad g(x) = x^2$$

Then:

$gf(x) = g(f(x)) = (x+3)^2$	$fg(x) = x^2 + 3$
$x \rightarrow f(x) \rightarrow g(f(x))$	$x \rightarrow g(x) \rightarrow f(g(x)$

range of 1st function = domain of 2nd function
$\Rightarrow$ domain of 1st function restricted by range of 2nd function

Eg:

$$f(x) = x + 3, \; x \in \mathrm{R} \qquad g(x) = \sin^{-1} x, \; -1 \leq x \leq 1$$

Domain of g = range of $f \Rightarrow -4 \leq x \leq -2$

Inverse functions

- Only exist for one to one functions

Eg:

$$f(x) = x + 5 \Leftrightarrow f^{-1}(x) = x - 5$$

$$f(x) = 3x + 2 \Leftrightarrow f^{-1}(x) = \frac{x-2}{3}$$

$$f(x) = \sqrt{x} + 1 \Leftrightarrow f^{-1}(x) = (x-1)^2$$

And:

$$ff^{-1}(x) \equiv x$$

- The range of f is the domain of f^{-1}.
- Inverse functions are reflections of the original functions in the line $y = x$.
- Specific values where $f = f^{-1}$ will always occur on the line $y = x$.
  Therefore, to find these points solve $f(x) = x$.

For a function to be a 'self inverse function' then:

$$f = f^{-1}$$

Eg:

$$f(x) = \frac{4}{x} \Leftrightarrow f^{-1}(x) = \frac{4}{x}$$

$$f(x) = 5 - x \Leftrightarrow f^{-1}(x) = 5 - x$$

- Self inverse functions are symmetrical about the line $y = x$.