

## Differential Equations

Process is...

1. Prepare for integrating by rearranging to get all  $x$  terms and  $y$  terms on separate sides
2. Integrate both sides
3. Combine the  $c$ 's and rearrange into format  $y = \dots$  THIS IS THE GENERAL SOLUTION
4. Use the coordinate given to find THE PARTICULAR SOLUTION

E.G. (Ex11E Qu2i)

Find the particular solution...

$$x(x-1)\frac{dy}{dx} = y$$

$$y = 1, x = 2$$

Rearrange

$$\int \frac{1}{y} dy = \int \frac{1}{x(x-1)} dx$$

Integrate both sides

Note that

$$\frac{1}{x(x-1)} = \frac{A}{x} + \frac{B}{(x-1)}$$

$$\frac{1}{x(x-1)} = \frac{A(x-1)}{x(x-1)} + \frac{Bx}{x(x-1)}$$

$$1 = A(x-1) + Bx$$

$$x = 0 \Rightarrow A = -1$$

$$x = 1 \Rightarrow B = 1$$

$$\int \frac{1}{x(x-1)} dx = \int \frac{-1}{x} dx + \int \frac{1}{(x-1)} dx$$

$$\int \frac{1}{y} dy = \int \frac{-1}{x} dx + \int \frac{1}{(x-1)} dx$$

$$\ln y = -\ln x + \ln(x-1) + c$$

...continues

Combine and rearrange

$$\begin{aligned}y &= e^{-\ln x + \ln(x-1) + c} \\y &= A \cdot e^{\ln x^{-1}} \cdot e^{+\ln(x-1)} \\y &= A \cdot e^{\ln\left(\frac{1}{x}\right)} \cdot e^{\ln(x-1)} \\y &= A \cdot \frac{1}{x} \cdot (x-1)\end{aligned}$$

$$y = \frac{A(x-1)}{x} \text{ THIS IS THE GENERAL SOLUTION}$$

Use coordinate to find particular solution

$$\begin{aligned}(2, 1) &\Rightarrow 1 = \frac{A(2-1)}{2} \\A &= 2\end{aligned}$$

$$y = \frac{2(x-1)}{x} = \frac{2x-2}{x} = 2 - \frac{2}{x} \text{ THIS IS THE PARTICULAR SOLUTION (any of these formats)}$$