Chain, Product, Quotient Rule Recap

Differentiate these.

Chain Rule

$y = (3x+4)^3$	$y = sin(2x)$	$y = e^{2x}$
$y = (3x^3+1)^3$	$y = sin(x^2)$	$y = e^{(x^3)}$
$y = \frac{4}{4-x^4}$	$y = \frac{1}{\left(\frac{1}{cosecx}\right)}$	$y = ln\left(\frac{1}{e^x}\right)$
$y = \sqrt{\sqrt{x}+1}$	$y = sin(cos(tanx))$	$y = e^{e^x}$

Product Rule

$y = (3x+4)(2x-3)$	$y = x^2 sinx$	$y = xe^x$
$y = (3x^2+4)(2x^4+3)$	$y = sinxcosx$	$y = x^3e^x$
$y = \sqrt{3x}\sqrt{2x}$	$y = sinxcosxtanx$	$y = xlnx$
$y = \left(\frac{x}{3}\right)^3 \sqrt[3]{x}$	$y = sin(x^2)cos(x^2)$	$y = e^{x^2} ln(x^2)$

Quotient Rule

$y = \frac{x+1}{2x+1}$	$y = \frac{3x^2}{sinx}$	$y = \frac{2x}{e^x}$
$y = \frac{x^2+1}{x^2-1}$	$y = tanx$	$y = \frac{e^x}{e^{-x}}$
$y = \frac{x^2}{\sqrt{x}}$	$y = \frac{x^2}{tanx}$	$y = \frac{e^{-4x}}{4e^{4x}}$
$y = \sqrt[3]{\frac{x+1}{x-1}}$	$y = \frac{cotx}{2secx}$	$y = \frac{ln(x^2)}{e^{x^2}}$

Chain, Product, Quotient Rule Recap - Answers

Differentiate these.

Chain Rule

$y=(3x+4)^3$ $\frac{dy}{dx}=9(3x+4)^2$	$y=sin(2x)$ $\frac{dy}{dx}=2cos(2x)$	$y=e^{2x}$ $\frac{dy}{dx}=2e^{2x}$
$y=(3x^3+1)^3$ $\frac{dy}{dx}=3(3x^3+1)^2(9x^2)$ $=27x^2(3x^3+1)^2$	$y=sin(x^2)$ $\frac{dy}{dx}=2xcos(x^2)$	$y=e^{(x^3)}$ $\frac{dy}{dx}=3x^2e^{(x^3)}$
$y=\frac{4}{4-x^4}=4(4-x^4)^{-1}$ $\frac{dy}{dx}=-4(4-x^4)^{-2}\times -4x^3$ $=\frac{16x^3}{(4-x^4)^2}$	$y=\frac{1}{\left(\frac{1}{cosecx}\right)}=\frac{1}{sinx}$ $=(sinx)^{-1}$ $\frac{dy}{dx}=-(sinx)^{-2}\times cosx$ $=\frac{-cosx}{(sinx)^2}=cotxcosecx$	$y=ln\left(\frac{1}{e^x}\right)$ $\frac{dy}{dx}=\frac{1}{\frac{1}{e^x}}\times -e^{-x}$ $=-e^x\times e^{-x}$ $=-e^0$ $=1$ (convince yourself that this is the case)
$y=\sqrt{\sqrt{x}+1}=\left(x^{\frac{1}{2}}+1\right)^{\frac{1}{2}}$ $\frac{dy}{dx}=\frac{1}{2}\left(x^{\frac{1}{2}}+1\right)^{-\frac{1}{2}}\times\frac{1}{2}x^{-\frac{1}{2}}$ $=\frac{1}{4}\times\frac{1}{\sqrt{\sqrt{x}+1}}\times\frac{1}{\sqrt{x}}$ $=\frac{1}{4\sqrt{x}\sqrt{\sqrt{x}+1}}$	$y=sin\big(cos(tanx)\big)$ $\frac{dy}{dx}=-cos\big(cos(tanx)\big)sin(tanx)sec^2x$	$y=e^{e^x}$ $\frac{dy}{dx}=e^{e^x}e^x=e^{e^x+x}$

Product Rule

$y = (3x+4)(2x-3)$ $\frac{dy}{dx} = 2(3x+4) + 3(2x-3)$ $= 12x - 1$	$y = x^2 sinx$ $\frac{dy}{dx} = x^2 cosx + 2xsinx$	$y = xe^x$ $\frac{dy}{dx} = xe^x + e^x$ $= (x+1)e^x$
$y = (3x^2+4)(2x^4+3)$ $\frac{dy}{dx} = 8x^3(3x^2+4) + 6x(2x^4+3)$ $= 2x[4x^2(3x^2+4) + 3(2x^4+3)]$ $= 2x[12x^4 + 16x^2 + 6x^4 + 9]$ $= 2x[18x^4 + 16x^2 + 9]$	$y = sinxcosx$ $\frac{dy}{dx} = cos^2x - sin^2x$	$y = x^3 e^x$ $\frac{dy}{dx} = 3x^2e^x + x^3e^x$ $= x^2e^x(x+3)$
$y = \sqrt{3x}\sqrt{2x}$ $= (6x^2)^{\frac{1}{2}} = \sqrt{6}x$ $\frac{dy}{dx} = \sqrt{6}$ (easy method?)	$y = sinxcosxtanx$ $= sinxcosx \times \frac{sinx}{cosx}$ $= sin^2x$ $\frac{dy}{dx} = 2sinxcosx$	$y = xlnx$ $\frac{dy}{dx} = lnx + 1$
$y = \left(\frac{x}{3}\right)^3 \sqrt[3]{x}$ $\frac{dy}{dx} = \left(\frac{x}{3}\right)^3 \times \frac{1}{3\sqrt[3]{x}} + \sqrt[3]{x} \times \frac{x^2}{3}$ $= \frac{x^3}{9\sqrt[3]{x}} + \frac{x^2\sqrt[3]{x}}{3}$ $= \frac{x^2}{3}\left(\frac{\sqrt[3]{x^2}}{3} + \sqrt[3]{x}\right)$	$y = sin(x^2)cos(x^2)$ $\frac{dy}{dx} = 2x[cos^2(x^2) + sin^2(x^2)]$	$y = e^{x^2} ln(x^2)$ $\frac{dy}{dx} = \frac{2e^{x^2}}{x} + 2xe^{x^2} ln(x^2)$

Quotient Rule

$y = \frac{x+1}{2x+1}$ $\frac{dy}{dx} = \frac{2x+1-2(x+1)}{(2x+1)^2}$ $= \frac{-1}{(2x+1)^2}$	$y = \frac{3x^2}{sinx}$ $\frac{dy}{dx} = \frac{6xsinx - 3x^2cosx}{sin^2x}$ $= \frac{3x}{sinx}\left(2 - \frac{x}{tanx}\right)$	$y = \frac{2x}{e^x}$ $\frac{dy}{dx} = \frac{2e^x - 2xe^x}{e^{2x}}$ $= \frac{2-2x}{e^x}$
$y = \frac{x^2+1}{x^2-1}$ $\frac{dy}{dx} = \frac{2x(x^2-1) - 2x(x^2+1)}{(x^2-1)^2}$ $= \frac{-4x}{(x^2-1)^2}$	$y = tanx = \frac{sinx}{cosx}$ $\frac{dy}{dx} = \frac{cos^2x + sin^2x}{cos^2x}$ $= 1 + sin^2x$ $= sec^2x$	$y = \frac{e^x}{e^{-x}} = e^x \times e^x = e^{2x}$ $\frac{dy}{dx} = 2e^{2x}$
$y = \frac{x^2}{\sqrt{x}}$ $\frac{dy}{dx} = \frac{2x\sqrt{x} - \frac{x^2}{2\sqrt{x}}}{x}$ $= 2\sqrt{x} - \frac{\sqrt{x}}{2}$	$y = \frac{x^2}{tanx}$ $\frac{dy}{dx} = \frac{2xtanx - x^2secx}{tan^2x}$ $= \frac{2x}{tanx} - \frac{x^2}{tan^2xcosx}$ $= \frac{x}{tanx}\left(2 - \frac{x}{tanxcosx}\right)$	$y = \frac{e^{-4x}}{4e^{4x}} = \frac{1}{4}(e^{-4x} \times e^{-4x})$ $= \frac{e^{8x}}{4}$ $\frac{dy}{dx} = 2e^{8x}$
$y = \sqrt[3]{\frac{x+1}{x-1}} = \frac{(x+1)^{\frac{1}{3}}}{(x-1)^{\frac{1}{3}}}$ $\frac{dy}{dx} = \frac{\frac{1}{3}(x-1)^{\frac{1}{3}}(x+1)^{\frac{-2}{3}} - \frac{1}{3}(x-1)^{-\frac{2}{3}}(x+1)^{\frac{1}{3}}}{(x-1)^{\frac{2}{3}}}$ $= \frac{1}{3}\left[\frac{(x-1)^{-\frac{1}{3}}}{(x+1)^{\frac{2}{3}}} - \frac{(x+1)^{\frac{1}{3}}}{(x-1)^{\frac{4}{3}}}\right]$ $= \frac{1}{3}\left[\frac{(x-1)-(x+1)}{(x+1)^{\frac{2}{3}}(x-1)^{\frac{4}{3}}}\right]$ $= \frac{-2}{3}\left[\frac{1}{\sqrt[3]{(x+1)^2(x-1)^4}}\right]$	$y = \frac{cotx}{2secx} = \frac{cosx}{sinx} \div \frac{2}{cosx}$ $= \frac{cosx}{sinx} \times \frac{cosx}{2}$ $= \frac{cos^2x}{2sinx}$ $\frac{dy}{dx} = \frac{-4sin^2xcosx - 2cos^3x}{4sin^2x}$ $= -xcosx - \frac{cosx}{2tan^2x}$	$y = \frac{ln(x^2)}{e^{x^2}}$ $\frac{dy}{dx} = \frac{\frac{2e^{x^2}}{x} - 2xe^{x^2}lnx^2}{e^{2x^2}}$ $= \frac{\frac{2}{x} - 2xlnx^2}{e^{x^2}}$